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Interval type-2 hesitant fuzzy set and its application in multi-criteria decision making ☆

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ABSTRACT

This paper proposes the concept of interval type-2 hesitant fuzzy set (IT2HFS), which is a generalization of interval type-2 fuzzy set and hesitant fuzzy set. Firstly, IT2HFS is defined, and its operation laws and corresponding properties are discussed. Then, its score function and aggregation operators are also outlined. Moreover, a multi-criteria decision making method for interval type-2 hesitant fuzzy problems is suggested based on the aggregation operators and score function. Finally, an example is illustrated to demonstrate the validity and feasibility of the proposed method.

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1. Introduction

Since fuzzy set (FS) was proposed by Zadeh (1965), it has been highly successful in various fields. A common problem when dealing with real-world multi-criteria decision making (MCDM) cases is that the criteria weights and criteria values of alternatives may be inaccurate, uncertain or incomplete due to the fuzziness and uncertainty of decision making problems. To manage vague information more efficiently, researchers have extended FSs to different forms, such as interval-valued fuzzy set (IVFS) (Gorzałczany, 1987; Turksen, 1986), hesitant fuzzy set (HFS) (Torra, 2010) and type-2 fuzzy set (T2FS) (Zadeh, 1975). Out of several higher order FSs, intuitionistic fuzzy sets (IFSs), which were introduced by Atanassov (1986), have been found to be highly useful when dealing with vagueness. Gau and Buehrer (1993) proposed the concept of vague set (VS), which is another generalization of FS. However, Bustince and Burillo (1996) pointed out that the notion of VS is the same as that of IFS. Some new techniques (Chen & Tan, 1994; Hong & Choi, 2000; Hu, Chen, & Chen, 2014; Liu & Wang, 2007; Wang, Zhou, Li, Zhang, & Chen, 2014; Ye, 2007, 2009, 2010) have been presented for solving fuzzy MCDM problems based on VSs theory or IFSs theory, where the characteristics of the alternatives are represented by VSs or IFSs. Wang and Zhang (2009) proposed intuitionistic trapezoidal fuzzy numbers along with their operators, which extended intuitionistic triangular fuzzy numbers. IVFS, which is a well-known generalization of ordinary FS, was introduced by Gorzałczany (1987) and Turksen (1986). Subsequently, Atanassov and Gargov (1989) introduced interval-valued intuitionistic fuzzy sets (IVIFSs) on the basis of both IFSs and IVFSs.

In reality, the exact membership function of FS is often difficult to determine and type-1 fuzzy sets (T1FSs) cannot really solve this problem very well. For example, it may be stated that the probability that a man's age is about 60 is possibly 0.8, but this cannot be expressed by an exact membership which T1FS can provide. To deal with this issue, Zadeh (1975) proposed T2FSs which are an extension of T1FSs. T2FSs, which are three dimensional, use both primary and secondary membership to provide more degrees of freedom and flexibility. Therefore, T2FSs have the advantage of modeling uncertainty more accurately when compared with T1FSs. However, the computational burden is considerable when using T2FSs to solve problems (Li & Gu, 2008). IT2FS can be viewed as a special case of general T2FS where all the values of secondary membership are equal to 1. Thus, it not only represents uncertainty better than T1FSs do, but also simplifies the computation processes when compared with T2FSs. To date, studies on IT2FSs can be categorized into two aspects. One aspect is theoretical research. For example, some basic definitions of IT2FSs have been proposed by Mendel, John, and Liu (2006). Moreover, there are some studies on the similarity of IT2FSs (Mitchell, 2005; Zeng & Li, 2006) which proposed some design methods for calculating this similarity. Furthermore, Wu and Mendel (2008) proposed a new method

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which can transform IT2FSs more efficiently. Hu, Zhang, Chen, and Liu (2013) proposed an MCDM method based on possibility degree of interval type-2 fuzzy number. The other aspect is the application of IT2FSs in the real world. A new approach based on interval type-2 fuzzy logic system was proposed by Shu, Liang, and Gao (2008) to analyze and estimate the network lifetime for wireless sensor networks. Linda and Manic (2011) developed a fuzzy voter design for fault tolerant systems on the base of IT2FSs.

However, when defining the membership degree of an element to a set, the difficulty of establishing the membership degree is not because there is a margin of error (as in IFS or IVFSs), or some possibility distribution (as in T2FS) on the possible values, but because there is a set of possible values (Torra, 2010). To deal with such cases, another generalization of FS, HFS which allows the membership degree to have a set of possible values was introduced by Torra and Narukawa (2009) and Torra (2010). Recently, some researches on HFSs have been conducted due to their superiority in expressing uncertainty. For example, Yu, Wu, and Zhou (2011) proposed an MCDM method based on Choquet integral under hesitant fuzzy environment. Furthermore, Wei (2012) developed some prioritized aggregation operators for aggregating hesitant fuzzy information, and applied them to develop some models for hesitant fuzzy MCDM problems in which the criteria are in different priority levels. Wei, Wang, Zhao, and Lin (2014) proposed the concept of the hesitant triangular fuzzy set based on HFS to permit the membership to have a set of possible triangular fuzzy numbers. Rodriguez, Martinez, and Herrera (2012) proposed the concept of hesitant fuzzy linguistic term set (HFLTS) under hesitant fuzzy environment and different computational functions and properties of HFLTS have been developed to demonstrate its usefulness of the HFLTS in decision making. Lee and Chen (2013) presented a new fuzzy decision making method based on the likelihood-based comparison relations of HFLTSs. Rodríguez, Martinez, and Herrera (2013) extended the HFLTS to the group decision environment and proposed the corresponding group decision making method.

However, the current methods, which operate under the hesitant environment, only assume that the membership has a set of possible crisp or exact type-1 fuzzy numbers (T1FNs). In a real-world situation, even a set of possible crisp and exact values are inefficient and inadequate when modeling a real-life decision problem due to the increasing complexity of the socioeconomic environment and the vagueness of inherent subjective nature of human beings. To date, many studies have been conducted that extend HFSs (Peng, Wang, Wang, Yang, & Chen, 2015; Wang, Wang, Chen, Zhang, & Chen, 2014; Wang, Wu, Wang, Zhang, & Chen, 2015; Zhang & Xu, 2014; Zhang, Wang, Tian, & Li, 2014) and IT2FSs (Gong, Hu, Zhang, Liu, & Deng, 2015; Ngan, 2013; Wang, Chen, Zhang, Chen, & Wang, 2015; Wang, Yu, et al., 2015) respectively to grasp the uncertainty and fuzziness of incomplete and inaccurate information more efficiently. But until now, no studies have been focused on the combination of HFSs and IT2FSs. Thus, we propose the concept of IT2HFS which is an extension of IT2FS under the hesitant fuzzy environment. It is common for decision makers (DMs) to obtain linguistic information, yet it is difficult to collect IT2FN data directly. In this paper, considering the fact that linguistic information can be transformed into IT2FN data, the alternative criterion ratings and the criterion importance weights are transformed into linguistic variables to overcome the difficulty of data collection.

On one hand, when interval type-2 fuzzy information is collected from a group of DMs, the collection is indeed an IT2HFS; on the other hand, when the linguistic terms in HFLTS are turned into IT2FNs, HFLTS is transformed into IT2HFS. Compared with the general type-1HFS, IT2HFS has the following advantages. Firstly, IT2HFS can grasp the uncertainty and fuzziness of incomplete and inaccurate information, which contains a set of IT2FSs characterized by primary and secondary membership, more efficiently. Secondly, IT2HFS provides an indirect way to deal with the HFLTS, because the latter can be transformed into IT2HFS in some MCDM problems, but HFLTS cannot be handled in a direct way when evaluating information is given in the form of HFLTS. Thirdly, the use of IT2HFS can simplifies the computation process of HFLTS, as the computational burden of the latter is heavy. We propose the concept of IT2HFS to enrich the theory of HFS and overcome some of their limitations.

In this paper, by proposing the concept of IT2HFSs based on HFSs and IT2FSs, HFLTSs is transformed into IT2HFSs, which permit a set of IT2FSs to represent human's preference information. Based on the definition of IT2HFS, its operations and score function of IT2HFS are proposed. Then, the aggregation operators of IT2HFS are introduced to obtain the overall value of alternatives. Finally, on the basis of the proposed new score function and aggregation operators in this paper, the ranking result of a set of alternatives is obtained and the best one is selected.

The rest of this paper is arranged as follows. In Section 2, some basic concepts of HFSs and IT2FSs are briefly reviewed respectively. Then, the concept and operation laws of IT2HFS are proposed in Section 3. Furthermore, some corresponding properties are proved and a new score function of IT2HFS is also proposed in this section. Additionally, in Section 4 some aggregation operators of IT2HFS are developed, which form the basis of MCDM method with interval type-2 hesitant fuzzy information. In Section 5, the aggregation operators of IT2HFS are applied to MCDM problems with interval type-2 hesitant fuzzy information and a final ranking result is obtained based on the proposed score function. In Section 6, an illustrative example and a comparative analysis are proved to demonstrate the feasibility and advantage of the proposed method. Finally, conclusions are drawn in Section 7.

2. Preliminaries

2.1. Hesitant fuzzy set

The concept of HFSs was proposed by Torra (2010) and Torra and Narukawa (2009), to deal with the problems where membership of an element to a given set includes several different values. In this section, we briefly describe some basic concepts and basic operation laws related to HFSs.

Definition 1 (*Torra, 2010; Torra and Narukawa, 2009*). Let X be a reference set, a HFS on X is defined in terms of a function h that returns a subset of [0, 1].

To make it understood easily, a HFS can be represented by a mathematical symbol:

$$M = \{< x, h_M(x) > | x \in X\},$$

where $h_M(x)$ is a set of some values in [0,1], denoting the possible membership degrees of the element $x \in X$ to the set M. For convenience, Xia and Xu (2011) call $h = h_M(x)$ a hesitant fuzzy element (HFE) and H the set of all HFEs.

To compare HFEs, Xia and Xu (2011) proposed a score function as follows.

Definition 2 Xia and Xu, 2011. For a HFE $h, s(h) = \frac{1}{\#h}\sum_{\gamma \in h}\gamma$ is called the score function of h, where $\#h$ is the number of the elements in h.

For two HFEs h_1 and h_2, if $s(h_1) > s(h_2)$, then $h_1 > h_2$; if $s(h_1) = s(h_2)$, then $h_1 = h_2$.

Furthermore, Xia and Xu (2011) defined some operations on the HFEs h, h_1 and h_2:

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