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A method for discrete stochastic MADM problems based on the ideal and nadir solutions ☆

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ABSTRACT

Many real life decision making problems can be modeled as discrete stochastic multi-attribute decision making (MADM) problems. A novel method for discrete stochastic MADM problems is developed based on the ideal and nadir solutions as in the classical TOPSIS method. In a stochastic MADM problem, the evaluations of the alternatives with respect to the different attributes are represented by discrete stochastic variables. According to stochastic dominance rules, the probability distributions of the ideal and nadir variates, both are discrete stochastic variables, are defined and determined for a set of discrete stochastic variables. A metric is proposed to measure the distance between two discrete stochastic variables. The ideal solution is a vector of ideal variates and the nadir solution is a vector of nadir variates for the multiple attributes. As in the classical TOPSIS method, the relative closeness of an alternative is determined by its distances from the ideal and nadir solutions. The rankings of the alternatives are determined using the relative closeness. Examples are presented to illustrate the effectiveness of the proposed method. Through the examples, several significant advantages of the proposed method over some existing methods are discussed.

1. Introduction

Multi-attribute decision making (MADM) methods have been applied to a wide range of real-world problems. Most of these methods appeared in the literature focus on cases where attribute values are crisp numbers or fuzzy numbers (Fan, Ma, Jiang, Sun, & Ma, 2006;Hwang & Yoon, 1981; Jiang, Fan, & Ma, 2008). MADM problems usually have stochastic attributes, i.e., attributes that are stochastic or random variables. Consistent with the convention in the literature of stochastic MADM, the term "stochastic variables" rather than "random variables" will be used in this article. There are many such examples in real life. In a forest site productivity evaluation problem, ecological interpretability is a stochastic variable (Chuu, 2009). In decision support for investing in a potential industry, the environment conditions are stochastic variables (Zhang, Fan, & Liu, 2010). In the selection of the most desirable computer development project, the chance of success is a stochastic variable (Nowak, 2004). In the selection of a site for a waste treatment facility, transportation cost is a stochastic variable (Lahdelma, Salminen, & Hokkanen, 2002). In the selection of a strategic decision support model for a retailer's operation, market share is a stochastic variable (Sarker & Quaddus, 2002). In the formation of a management strategy for a forest ecosystem, net income from timber cuttings during the planning period is a stochastic variable (Lahdelma & Salminen, 2009). Hence, the development of MADM methods with the capacity of handling stochastic attributes has attracted the attention of many researchers.

Some approaches have been proposed to solve stochastic MADM problems from different perspectives (Fan, Liu, & Feng, 2010). Keeney and Raiffa (1976) initially proposed a method based on multi-attribute utility theory (MAUT) for dealing with MADM problems under uncertainty. Martel and D'Avignon (1982) and Martel, D'Avignon, and Couillard (1986) aggregated evaluations of multiple experts to obtain random evaluations, called distributive evaluations, on the alternatives. Two indices, a confidence index and a doubt index, are calculated using these evaluations. A degree of credibility is obtained by combing the two indices. A fuzzy outranking relation characterized by the degree of credibility is used to capture the preferences of one alternative over another. D'Avignon and Vincke (1988) considered the preference indices given by the decision maker (DM), and proposed a multi-attribute

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procedure to aggregate random evaluations of the alternatives into random preference degrees.

Stochastic dominance (SD) rules have been used to solve the stochastic MADM problem. Martel and Zaras (1995) used SD rules and utility functions to determine the outranking relations between alternatives on each attribute. Based on these outranking relations, they used the ELECTRE method (Roy, 1985, 1991) to obtain the rankings of the alternatives. Zaras (1999) used a rough set approach for obtaining a set of decision rules. Based on these rules, the non-redundant set of attributes is identified. By applying multi-attribute SD rules to the reduced set of attributes, the rankings of the alternatives are obtained. Zaras (2001) combined the SD rules and the rough set approach to study the MADM problem with deterministic and stochastic evaluation information. Rough set is used to reduce the size of the attribute set. Multi-attribute SD rules defined by Zaras (2001) are used to determine the dominance relations of alternatives on the smaller set of attributes. Zaras (2004) studied the MADM problem with deterministic, fuzzy and stochastic evaluation information. He proposed mixed-data multi-attribute dominance to identify the preference relation between alternatives on each attribute. Based on the dominance relations, several decision rules are generated by using the rough set approach and the rankings of the alternatives are obtained subsequently. For MADM problems with stochastic information, Nowak (2004) used SD rules to determine the dominance relationship associated with a pair of alternatives, and then identified strict preference, weak preference and indifference preference between alternatives on a single attribute. The rankings of the alternatives are then obtained by using the ELECTRE-III distillation procedures (Roy, 1985, 1991). Nowak (2007) studied the stochastic MADM problem using the DM's aspiration information. The number of alternatives is progressively reduced according to the DM's aspiration threshold. Furthermore, he used the SD rules to select the desirable alternative from the reduced set of alternatives. Zhang et al. (2010) introduced the concept of SD degree (SDD) to measure the strength of dominance of one alternative over another. Based on the overall SDD matrix, the rankings of the alternatives are obtained by using PROMETHEE-II (Brans & Vincke, 1985; Kolli & Parsaei, 1992).

In addition, other methods for solving stochastic MADM problems have been reported in the literature. Văduva and Resteanu (2009) examined a MADM problem with stochastic attribute values. They first standardized the stochastic attribute values. The standardized stochastic attribute values are then transformed into Shannon's entropy or Onicescu's informational energy. By using a simple additive weighting approach or using TOPSIS (Technique for Order Preference by Similarity to Ideal Solution), the Shannon's entropy or Onicescu's informational energy of each alternative is aggregated to obtain the rankings of all alternatives. Fan et al. (2010) proposed a method based on pairwise comparisons of alternatives with random evaluations to solve stochastic MADM problems. After computing superior, indifferent and inferior probabilities on pairwise comparisons of alternatives, the rankings of all alternatives are obtained. Fan, Zhang, Liu, and Zhang (2013) proposed a method based on the ideal and anti-ideal points for the stochastic MADM problem, where consequences of alternatives with respect to attributes are represented by stochastic variables with cumulative distribution functions. In this current study, the term "nadir solution" or "nadir point" instead of "anti-ideal point" is used. Ideal solutions are also called positive ideal solutions and nadir solutions are also called negative ideal solutions in the literature.

Prior studies have significantly enriched the theories and techniques of stochastic MADM problems. However, there are still limitations with these existing methods. For example, in methods using MAUT, the utility function is often difficult to obtain (Nowak, 2004). In methods using confidence indices and preference indices, the meanings of these indices are sometimes not easily interpretable (Stewart, 2005, chap. 11). Methods based on SD rules sometimes cannot determine or identify the dominance relation between two distinct alternatives (Leshno & Levy, 2002). When the dominance relation cannot be established, the rankings of the alternatives cannot be determined. In methods using TOPSIS, the stochastic attribute values are transformed into crisp or interval values. Obviously, this transformation causes information loss. In methods using ideal and anti-ideal points, the cumulative distribution functions of discrete stochastic variables are obtained to determine the ideal and anti-ideal points. However, the probabilities of the ideal and anti-ideal points at some values may be different from those of the stochastic attribute values. Hence, the probability distributions of ideal and anti-ideal points may not be closely related to those of the stochastic attribute values. However, in the classical TOPSIS method, the ideal and anti-ideal points are used to measure the preference relations using the attribute values. Hence, it is valuable to develop a novel method for solving stochastic MADM problems so as to overcome the above limitations.

Inspired by the use of ideal and nadir solutions in the TOPSIS method, this study provides a new way of ranking the alternatives when the attributes are discrete stochastic variables. The proposed method is motivated by the following ideas. Just like in the TOPSIS method, the chosen alternative should have the shortest distance from the ideal solution and the longest distance from the nadir solution, if possible. The method should overcome some of the limitations in the existing methods as mentioned above. Furthermore, this method should not need much judgmental input from the users such as the confidence indices, preference indices and/or utility functions in some existing methods, and should not need to verify the SD relations.

The developed method has three major components. Using SD relations, an ideal variate and a nadir variate for a set of stochastic variables are defined first. The ideal and nadir variates are defined on a single attribute. For stochastic MADM problems, the ideal solution is a vector of ideal variates and the nadir solution is a vector of nadir variates. A metric measuring the distance between two discrete stochastic variables is then defined. Using the ideal and nadir solutions and the metric, the method for stochastic MADM problems is finally developed. In the method, the probability distributions of the ideal and nadir solutions are determined first, the distances of each alternative from the ideal and the nadir solutions are then calculated, and the relative closeness of each alternative is finally calculated by using these distances. The relative closeness of the alternatives is then used to obtain the rankings of the alternatives.

The rest of this paper is organized as follows. A brief introduction to the classical TOPSIS method is given in Section 2. SD relations, ideal and nadir variates, and stochastic expectations are discussed in Section 3. A metric measuring the distance between two discrete stochastic variables is presented in Section 4. The method based on the ideal and nadir solutions to solve the stochastic MADM problem is presented in Section 5. Two examples illustrating the feasibility and effectiveness of the proposed method are presented in Section 6. Summaries and conclusions are given in Section 7.

2. A brief introduction to the classical TOPSIS method

Originally proposed by Hwang and Yoon (1981), the TOPSIS method is an effective tool for dealing with MADM problems (Awasthi, Chauhan, Omrani, & Panahi, 2011; Dymova,