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# Preference relations based on hesitant-intuitionistic fuzzy information and their application in group decision making ☆

Wei Zhou [a,b], Zeshui Xu [b,*], Minghui Chen [a]

[a] International Business School, Yunnan University of Finance and Economics, Kunming 650221, PR China
[b] Business School, Sichuan University, Chengdu 610064, PR China

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ABSTRACT

Preference relations are a powerful quantitative decision approach that assists decision makers in expressing their preferences over alternatives. In real-life applications, decision makers may not be able to provide exact preference information with crisp numbers. To solve this problem, a hesitant-intuitionistic fuzzy number (Hesitant-IFN) is proposed in this paper, and a proposal for the hesitant-intuitionistic fuzzy preference relation (Hesitant-IFPR) and its complementary form (Hesitant-IFCPR) for uncertain preference information are presented. Compared with other preference relations, the proposed relations use hesitant fuzzy elements (HFEs) to express the priority intensities of decision makers and produce the corresponding non-priority intensities by a conversion formula. In addition, we have deduced the operational laws and comparative methods of Hesitant-IFNs and used such information to investigate the corresponding aggregation operators and the approximate consistency tests. Next, we have constructed a group decision-making approach under a hesitant-intuitionistic fuzzy environment. Finally, two case studies are presented to illustrate the preference relations, the approximate consistency tests and the group decision method.



## 1. Introduction

Because of the high complexity of socioeconomic environments, it is difficult to acquire sufficient statistical data for practical decision making. Thus, the preference relation, which is a key analysis technique, has attracted wide-spread attention in the field of decision making and produced accurate results in recent few decades (Çakır, 2008; Jiang, Xu, & Yu, 2013; Lee, 2012; Orlovsky, 1978; Ovchinnikov & Roubens, 1991; Saaty, 1977; Wang & Fan, 2007; Xu, 2011; Yu, Xu, & Liu, 2013; Zhang & Xu, 2014; Zhukovin, Burshtein, & Korelov, 1987).

The analytic hierarchy process (AHP) is a classical quantitative decision approach based on the preference relations (Saaty, 1977, 1980), and it has been successfully used to support decisions in numerous business environments; the process expresses the decision-maker's preference information over alternatives through pairwise comparisons with crisp numbers. However, because of the complexity and uncertainty of real problems that require decisions, time pressure, lack of knowledge, and the decision maker's limited expertise, it is impracticable for the decision makers to use exact values to express their preference information for all alternatives. To address this issue, extended preference relations, such as the interval-valued fuzzy preference relations (Bustince & Burillo, 2002; Xu, 2013), the triangular fuzzy preference relations (Xu, 2002), the incomplete interval fuzzy preference relations (Xu, 2007a; Xu, Li, & Wang, 2014; Wang, 2014), and the interval-valued multiplicative preference relations (Liu, 2009), have been proposed to allow the decision makers to utilize different types of fuzzy numbers to express their preference information.

Basic elements of these preference relations only provide the priority intensity, which indicates that an alternative is preferred over another and tends to overlook the corresponding non-priority information. To solve this problem, the intuitionistic fuzzy preference relations (Szmidt & Kacprzyk, 2003; Xu, 2007b; Gong, Li, Zhou, & Yao, 2009; Gong, Li, Forrest, & Zhao, 2011) were developed. Recently, Xia and Xu (2011b), Xia, Xu, and Liao (2013) and Xia and Xu (2013) constructed the additional intuitionistic multiplicative preference relations and hesitant fuzzy multiplicative preference relations based on the Saaty's 1–9 scale, and Jiang et al. (2013) studied their compatibility measures and consensus models.

Based on the aforementioned technological analysis, fuzzy preference relations have been developed to avoid using exact values to express preference information and intuitionistic fuzzy preference relations (IFPRs) have been proposed to introduce

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* Corresponding author. Tel.: +86 25 84483382.
*E-mail addresses:* zw453@163.com (W. Zhou), xuzeshui@263.net (Z. Xu).

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non-priority intensity and effectively describe fuzziness and uncertainty, thus indicating that there are different and distinct advantages in the two types of extended preference relations. This study has synthesized these advantages by introducing a more general fuzzy number hesitant fuzzy element (HFE) and proposing the hesitant-intuitionistic fuzzy number (Hesitant-IFN). The corresponding Hesitant-IFPR and complementary form Hesitant-IFCPR have also been developed, and their basic operations, comparative laws, aggregation operators and approximate consistency test have been studied in detail. By using these calculations and theories, we have constructed a group decision-making approach under the Hesitant-IFN environment and provided two case studies as illustration.

This paper is organized as follows: the Hesitant-IFN is proposed in Section 2, and in Section 3, we further develop the Hesitant-IFPR and the Hesitant-IFCPR and introduce an aggregation operator. In Section 4, we provide an approximate consistency test and present the corresponding group decision-making approach. Two case studies are discussed in Section5, and conclusions are included in Section 6.

## 2. Hesitant-IFN and its operations

In this section, we briefly review two basic concepts of IFN and HFE and their operational laws. Next, we introduce the Hesitant-IFN and investigate its operational laws.

### 2.1. Some basic concepts

To extend the fuzzy set (Zadeh, 1965), an intuitionistic fuzzy set (A-IFS or IFS) that provides a powerful tool to address vagueness has been proposed (Atanasov, 1986). A prominent characteristic of the IFS is the simultaneous consideration of membership degree and non-membership degree. The concepts of IFS are as follows:

**Definition 1** (Atanasov (1986)). If $X = (x_1, x_2, \ldots, x_n)$ is fixed, then an IFS $A$ in $X$ can be defined as follows:

$$A = \{(x_i, \mu(x_i), v(x_i)) | x_i \in X\} \tag{1}$$

where $\mu(x_i) \in [0,1]$ and $v(x_i) \in [0,1]$ satisfy $0 \leqslant \mu(x_i) + v(x_i) \leqslant 1$ for all $x_i \in X$ and $\mu(x_i)$ and $v(x_i)$ are the membership degree and non-membership degree, respectively, of the element $x_i \in X$ to $A$.

For computational convenience, Xu and Yager (2006) referred to $(\mu_a, v_a)$ as an Atanassov intuitionistic fuzzy number (A-IFN or IFN) and denoted it as $a$ with the conditions $0 \leqslant \mu_a, v_a \leqslant 1$ and $0 \leqslant \mu(x_i) + v(x_i) \leqslant 1$.

**Definition 2** (Xu and Yager (2006)). For an IFN $(\mu_a, v_a)$, $s(a) = \mu_a - v_a$ and $h(a) = \mu_a + v_a$ are the score function and the accuracy function, respectively, of $a$. For two IFNs $a$ and $b$, if $s(a) < s(b)$, then $a < b$; and if $s(a) = s(b)$, then (i) if $h(a) = h(b)$, then $a = b$; (ii) if $h(a) < h(b)$, then $a < b$; (iii) if $h(a) > h(b)$, then $a > b$.

If $a = (\mu_a, v_a)$, $a_1 = \left(\mu_{a_1}, v_{a_1}\right)$ and $a_2 = \left(\mu_{a_2}, v_{a_2}\right)$ are three IFNs, and $\lambda > 0$, then the following operational laws are valid:

(1) $a_1 \oplus a_2 = \left(\mu_{a_1} + \mu_{a_2} - \mu_{a_1} \cdot \mu_{a_2}, v_{a_1} \cdot v_{a_2}\right)$;
(2) $a_1 \otimes a_2 = \left(\mu_{a_1} \cdot \mu_{a_2}, v_{a_1} + v_{a_2} - v_{a_1} \cdot v_{a_2}\right)$;
(3) $\lambda a = \left(1 - (1 - \mu_a)^{\lambda}, v_a^{\lambda}\right)$;
(4) $a^{\lambda} = \left(\mu_a^{\lambda}, 1 - (1 - v_a)^{\lambda}\right)$.

Recently, Torra and Narukawa (2009) proposed a hesitant fuzzy set (HFS), which is a more general fuzzy set and permits the membership to include a set of possible values.

**Definition 3** (Torra and Narukawa (2009)). If $X$ is a fixed set, then an HFS on $X$ is in terms of a function that, when applied to $X$, yields a subset of $[0,1]$.

To be easily understood, Xia and Xu (2011) expressed the HFS by a mathematical symbol:

$$E = \{\langle x, h_E(x)\rangle | x \in X\} \tag{2}$$

where $h_E(x)$ is a set of some values in $[0,1]$, denoting the possible membership degrees of the element $x \in X$ to set $E$, and represents HFE.

**Definition 4** (Xia and Xu (2011a)). For a HFE $h, s(h) = \sum_{\gamma \in h} \gamma / l(h)$ represents the score function of $h$, where $l(h)$ is the number of values in $h$ and $\gamma$ is the element of the hesitant fuzzy set $h$. For two HFEs $h_1$ and $h_2$, if $s(h_1) > s(h_2)$, then $h_1 > h_2$; and if $s(h_1) = s(h_2)$, then $h_1 = h_2$.

Moreover, the operational laws related to any three HFEs $h, h_1$ and $h_2$, are expressed as follows:

| | |
|---|---|
| (1) $h^{\lambda} = \cup_{\gamma \in h}\{\gamma^{\lambda}\}, \lambda > 0$; | (2) $\lambda h = \cup_{\gamma \in h}\left\{1 - (1-\gamma)^{\lambda}\right\}$, $\lambda > 0$; |
| (3) $h_1 \otimes h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2}\{\gamma_1 \gamma_2\}$; | (4) $h_1 \oplus h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2}$ $\{\gamma_1 + \gamma_2 - \gamma_1 \gamma_2\}$; |
| (5) $\lambda(h_1 \oplus h_2) = \lambda h_1 \oplus \lambda h_2$; | (6) $(h_1 \otimes h_2)^{\lambda} = h_1^{\lambda} \otimes h_2^{\lambda}$. |

Here, the union operator "$\cup$" is a basic operation of HFEs according to Xia and Xu (2011), and indicates a set of all calculated results that is consistent with the definition of HFE.

### 2.2. Hesitant-IFN and operational and comparative laws

The main characteristic of IFN is the simultaneous consideration of both membership and non-membership degrees, which is presented as $A = \{(x_i, \mu(x_i), v(x_i)) | x_i \in X\}$, and the advantage of HFE is the general presentation of the membership degree with a set of possible values, which is presented as $E = \{\langle x, h_E(x)\rangle | x \in X\}$. To synthesize these two fuzzy sets, we have developed the following hesitant-intuitionistic fuzzy set (Hesitant-IFS) and number (Hesitant-IFN):

**Definition 5.** If $X = (x_1, x_2, \ldots, x_n)$ is a fixed set, then a Hesitant-IFS $K$ on $X$ can be defined as follows:

$$K = \{(x_i, h_K(x_i), v_K(x_i)) | x_i \in X\} \tag{3}$$

where $h_K(x_i)$ and $v_K(x_i)$ represent the membership degree and the non-membership degree, respectively, of element $x_i$ to $K$, and $h_K(x_i)$ is a HFE with $h_K(x_i) \subseteq [0,1]$ and $\max\{h_K(x_i)\} + v_K(x_i) \leqslant 1$. In addition, the pair $(h_K(x_i), v_K(x_i))$ represents the Hesitant-IFN.

In Hesitant-IFN $K = \{(x_i, h_K(x_i), v_K(x_i)) | x_i \in X\}, h_K(x_i)$ is the membership degree, which is presented with HFEs, and $v(x_i)$ is the non-membership degree. Thus, this new fuzzy number could effectively synthesize the characteristics of IFN and HFE. To demonstrate this synthesized effect, we have presented their corresponding simplified forms and a simple example as follows: