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Multi-stage multi-objective production planning using linguistic and numeric data-a fuzzy integer programming model

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ABSTRACT

This paper introduces a methodology to solve a multi-stage production planning problem having multiple objectives, which are conflicting, non-commensurable and fuzzy in nature. The production process consists of multiple stages having one or more machines in each stage. Every processing stage produces work-in-process, semi-finished items as an output, which becomes an input to the subsequent stage either fully or partially depending on the cycle times of the machines. Events of machine breakdowns and imbalances in input–output relations in one or more stages may affect both work-in-process (WIP) and final production targets. Our paper provides a methodology based on fuzzy logic to maintain the desired balanced input–output relation at each stage and the targeted production output at the final stage. This is done by procurement of work-in-process inventory (WIP) and installation of new machines at appropriate stages. The objectives or goals expressed in linguistic terms are represented as fuzzy sets. The Induced Ordered Weighted Averaging (IOWA) operator is used to aggregate the objectives as per their priorities and finally to formulate the production process as a Mixed Integer Programming (MIP) problem. The solution to MIP shows the degrees of achievements of the production process objectives. The methodology is illustrated with a real life application of crankshaft productions in an automobile industry.

1. Introduction

The multi-stage production system having one or more machines at each stage takes inputs from the preceding stage and produce outputs that become inputs to the machines in the subsequent stage. Often the organizations face difficulties in its production process such as given below.

1. Imbalanced input–output relations between the consecutive machines may cause halt in the operation of the production line.
2. Existence of multiple numbers of objectives those are conflicting, non-commensurable, and fuzzy in the production planning process.
3. Random machine breakdowns may prevent the process to determine the machine capacity (in daily/monthly) in clear terms.
4. Work in process inventory (WIP) and new machine requirements in the production process stages are difficult to determine due to the unavailability of the breakdown data in clear terms. Among the problems mentioned above, imbalanced input–output relations and random machine breakdowns in the machine line system are most critical. The imbalanced input–output relationships in stages and randomness in machine breakdowns may lead to some amount of idle time in the machines and thereby affect the production target at the end. WIP creation or new machine installations (NMI) as stand by machines or both, at appropriate stages would work as a support for maintaining the balanced input–output relation and help in keeping the production line operational. Since NMI incurs a heavy cost to the company, this option is opted only when the existing machines are inadequate to deliver the required outputs or the WIP cannot be managed from the external sources at cheaper rates. Further, the amount of WIP creation and the number of new machine installations are dependent on the level of unbalanced input–outputs in the consecutive stages.

Our paper overcomes the above problems by modeling the production process as a Goal Programming problem. The following objectives, in decreasing order of importance, are considered to have an efficient production process.

Maximize production.
Maintain appropriate WIP in the machining stages.

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Minimize WIP inventory procurement cost.
Minimize new machine installations cost.

The above objectives are expressed in fuzzy terms due to the following reasons:

1. The random machine breakdowns make it difficult to know the capacity of the machines in exact terms. However, one can express them in vague or fuzzy terms. This precludes the determination of the machine outputs in each stage including the final stage (production target) in clear terms. The uncertainty in outputs in intermediate stages makes the management to express their production needs in fuzzily defined linguistic terms.
2. At times management may not be able to express their production target in a specific value because of lack of requisite information. In this situation, the management prefers to express the production and other WIP inventory targets in linguistic values. The linguistic values are defined appropriately in fuzzy terms or in fuzzy intervals.
3. In real life production system, at times the company is flexible enough to deviate from its target up to a certain limit given the high cost of the resources or inadequacy of their availability. This makes an analyst to express the targeted values in fuzzy terms.

Our work has introduced a methodology that addresses the fuzziness in the production process after incorporating the facts of unbalanced input output relations among the consecutive machines and the randomness in machine breakdowns. The financial, logistical and operational constraints related to WIP inventory and NMI, are also considered in our paper. Our paper considers the multiple numbers of objectives and their targets in linguistic terms as desired by the management. Our paper uses Induced Ordered Weighted Average (IOWA) aggregation operator (Yager, 2003) to aggregate the multiple numbers of objectives into a single objective. The order of importance of the objectives is maintained in the aggregation process. The following goals as expressed by the management, in decreasing order of their priorities are considered in our paper.

D.1. Production target should be 10,746. However, if required a *small* deviation is acceptable.
D.2. Work-in-process inventory be maintained ideally *around 10 units.* However, little deviation over 10 units is acceptable, but in no case, it should exceed 20 units at any stage.
D.3. Inventory holding *cost should be around half of its current value (6846).*
D.4. New machine installation cost *should be not be higher than 747.*

In the above, the italic words are fuzzy.

The fuzziness in the goal statements indicates management's flexibility in attainment of the goals. The fuzziness in goals is approximated to relevant linguistic terms in S to determine the allowed deviations numerically. The set *S* consists of nine basic linguistic terms (Fig. 1) in the form of fuzzy numbers to grasp the meaning of flexibility in the goal statements. The nine linguistic terms are shown below.

$$S = (s_0 = IMP, s_1 = NLG, s_2 = VL, s_3 = L, s_4 = M, s_5 = H, s_6 = VH, s_7 = SH, s_8 = EH).$$

Graphically, the fuzzy number representation of linguistic terms in *S* are shown in Fig. 1.

Representing the flexibility in the goals as linguistic 2-tuples in *S* helps us to represent the goals as fuzzy sets and define membership functions that are semantic to the allowable deviations. For example, the goal statement "*produce approximately 10,746 units*" can be interpreted, as the management does not want much deviation from the target 10,746. The deviation "*close to very low*" written as (VL, −0.1) from the targeted goal of 10,746 units can be interpreted as the deviation from target is so low that it is about 10% lower than the linguistic measure "very low". Following the methodology (Herrera & Martinez, 2000), the linguistic term set in Fig. 1 is used to obtain the numerical equivalent of '(VL, −0.1)' as 0.238. The linguistic value (VL, −0.1) or its numerical counterpart 0.238 represents the allowable deviation to the production target as per the company's choice. Thus, we have the least production level acceptable to the management as $(1 - 0.238) \times 10,746 = 8188$. Therefore, the management' acceptance of a production level 'P' can be defined through the membership function in Eq. (1) below.

$$\mu_P(P) = \begin{cases} 0, & \text{if } P < 8188, \\ \frac{P-8188}{2558}, & \text{if } 8188 \leqslant P < 10,746, \\ 1, & \text{if } P \geqslant 10746. \end{cases} \quad (1)$$

As shown in Eq. (1), the management is fully satisfied if the production is 10,746 units or more. The satisfaction level gradually decreases when the production units are below 10,746 and becomes zero when it becomes 8188 or less. Similarly, we can have membership functions of other goals through their goal statements and the deviations.

The motivating factors for using this technique in real life production planning problems are as follows.

(1) The multi-stage production process can have multiple objectives and the management may express their targets in day-to-day linguistic terms. The technique in our paper appropriately translates the linguistically defined targets to numeric terms in a range identified through fuzzy sets. (2) The methodology is applicable even if machine breakdown data is not available in exact terms. In such cases, the production rates are expressed after incorporating the breakdown data, in fuzzy terms. (3) The methodology incorporates the priority as perceived by the management by aggregating the objectives using IOWA. (4) Our work gives a generic model that can be applied to any type of multi-stage production planning problems. (5) The applicability of our methodology is illustrated by applying it to a crankshaft production problem in an automobile industry.

In the literature, multi-stage production process are studied in various dimensions such as design, planning, line balancing and inventory management (Afentakis, Garish, & Karmakar, 1984; Battaia & Dolgui, 2013; Blackburn & Millen, 1982; Deckro & Herbert, 1984; Gabby, 1979; Vickery & Markland, 1986). The methodology (Weng, 1998) describes the production control policy for multi-stage manufacturing system facing random demands and manufacturing uncertainties across the stages by deploying buffer capacity both statically and dynamically. Though the work is insightful in proving efficiency of dynamic buffer allocation under the assumption of homogeneity of resources across multiple stages, it does not account for some vital entities of the production process such as underlying cost of implementation and other logistic requirements. In Eynan and Dong (2012), a multi stage production planning is done with flexible capacity planning with an aim of cost minimization under the assumption of the availability of exact information. The exact information on different dimensions of the problem is rarely found. In Aidurgam and Elshafei (2012), a multi-stage production policy with the single objective to achieve minimization of a Taguchi-type quadratic loss function across all production stages is taken for targeting the process. The policy (Aidurgam & Elshafei, 2012) does not account for unequal cycle times among stages. In Hsiaoa, Lin, and Huang (2010), the problem