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Extended symmetric and asymmetric weight assignment methods in data envelopment analysis ☆

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ABSTRACT

Dual weight restrictions are commonly suggested as a remedy to the problem of low discriminatory power and absurd marginal prices in conventional Data Envelopment Analysis (DEA) models. However, weight restriction models also suffer from potential problems of infeasibility, lack of exogenous determination and ambiguous interpretations. The Symmetric Weight Assignment Technique (SWAT) addresses these concerns through a symmetric endogenous weight selection process. In this paper, we extend the SWAT method by proposing four new DEA models. Symmetric and asymmetric weights are rewarded and penalized, respectively, in the proposed models. The first model takes into account the symmetrical weights assigned to the outputs in the input-oriented model. The second model takes into account the symmetrical weights assigned to the inputs in the output-oriented model. The third and fourth models simultaneously take into account symmetric input–output weights in both the input and output orientations. We demonstrate the applicability of the proposed models and the efficacy of the procedures and algorithms with an application to Danish district heating plants.

1. Introduction

Any performance evaluation technique that results in a set of measures of lower dimensionality than the original production space must necessarily consider weighting the resources that are consumed and the outputs that are produced. The methodology for determining the relative costs or prices is one of the pivotal challenges in performance evaluation. Whereas market prices may be observed or elicited in certain circumstances, they may not necessarily reflect the social welfare effects due to externalities and horizon problems. Tradeoff rates may be inferred from preferences solicited from managers, although there is little incentive for managers to provide this information and it is likely to result in biased data. Engineering data may postulate costs for a given technology, but this may be doubtful in regulatory contexts as well as in the presence of technological innovation or process heterogeneity. Non-parametric frontier approaches such as the Data Envelopment Analysis (DEA) by Charnes, Cooper, and Rhodes (1978, 79), drawing on the seminal work of Farrell (1957)[1] address this issue by allocating sets of individual endogenous weights that put the individual unit in the best possible light. In this manner DEA provides the evaluator with a conservative performance estimate that is valid for a range of preference functions. Under a convex frontier specification, the analysis explicitly provides the evaluator with dual information that later may be used to refine the preference model of the evaluator by inserting additional constraints. In an open retrospective evaluation, where the modeling rests entirely at the discretion of the analyst or collectively of the units, such an

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[1] Whereas both Farrell (1957) and Charnes et al. (1978) are seminal contributions to the area of efficiency analysis and measurement, the viewpoints of the authors are different: Farrell defined a single-dimensional radial primal projection, Charnes et al. defined their metric from a productivity ratio-approach with an immediate dual approach in linear programming for the multi-dimensional case. Note that under the conventional assumptions for the productivity possibility set, the primal and dual approaches are equivalent under the duality theorem.

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approach may support organizational learning and development. Unrestricted weights are relevant in the determination of technical efficiency, i.e., the overall transformation rate of inputs to outputs irrespective of input costs and output preferences. However, the endogenously determined weights in conventional DEA may also lead to estimations of the efficient frontier that imply absurd cost functions, where certain inputs and outputs seemingly have no or extremely low costs and values, respectively. Consequently, the technical efficiency estimates are lower bounds to the "true" technical efficiency when taking into account a more constrained set of marginal costs and prices. The subsequent rankings are also weaker, leading to poor discriminatory ability for small data sets.

The use of weight restrictions techniques has a long tradition in the DEA literature (for the early development see e.g. the survey in Allen, Athanassopoulos, Dyson, & Thanassoulis (1997)). Golany (1988) developed a DEA model with ordinal relations among the weights for subsets of the inputs or outputs. Ali, Cook, and Seiford (1991) later proved that weak ordinal relations need non-standard DEA models of the type in Golany (1988). Dyson and Thanassoulis (1988) proposed a procedure for determining weight restrictions through direct constraints, their interpretation and derivation through linear regression. An alternative approach, restricting the weights through their relative shares, was first launched in Wong and Beasley (1990). Kornbluth (1991) extended the direct dual constraint approach by using the cone ratio constraint approach originally developed in Charnes, Cooper, Wei, and Huang (1989). Roll, Cook, and Golany (1991) proposed a DEA model where absolute bounds were imposed on the factor weights. Their method specified the eligible bounds for the weights as well as introducing the notion of a Common Set of Weights (CSW). Cook, Roll, and Kazakov (1990) and Roll et al. (1991) presented ratio-based approaches based on the dual "weight matrix" obtained in an unrestricted DEA model onto which specific interval coefficients were applied to derive acceptable bounds for the dual weight variation.

Thompson, Langemeier, Lee, Lee, and Thrall (1990) proposed the Assurance Region (AR) method for factoring weight control by setting ratios to improve the discriminatory power of DEA. Thompson, Lee, and Thrall (1992) studied the AR-efficiencies of US independent oil and gas producers using the DEA ratio and convex models in the presence of weight bounds. Roll and Golany (1993) proposed a framework involving a number of method for controlling the input and output weights in DEA by setting their bounds. They also classified the weight bounding methods in the literature. Cook, Kress, and Seiford (1992) proposed an alternative DEA method by imposing distinct conditions on the weights.

Allen et al. (1997) categorized the weight restriction methods into three categories: (1) the Assurance Region I (ARI) methods first developed by Thompson, Singleton, Thrall, and Smith (1986); (2) the Assurance Region II (ARII) methods first proposed by Thompson et al. (1990), often called linked-cone assurance region methods; and (3) the absolute weight restriction method first introduced by Dyson and Thanassoulis (1988). The AR methods are different from the absolute weight restrictions methods since the ratios between the weights are imposed to be within given bounds instead of imposing the weights to be within given bounds. The ARI methods determine the ratios between the input and the output weights separately while the ARII methods determine the ratios that connect the input weights to the output weights.

Halme and Korhonen (2000) considered two types of preference information involving the most preferred inputs and outputs and the information on the weights of the inputs and outputs in a DEA problem using the value efficiency analysis presented by Halme, Joro, Korhonen, Salo, and Wallenius (1999). Podinovski (2004a) first proposed a modified approach by incorporating the production trade-offs into the DEA models that maintained all the principle properties of efficiency, particularly, the radial target of each inefficient DMU. This method was then applied in the multiplier DEA models with weight restrictions. Podinovski (2004b) discussed the problem of using absolute weight bounds in DEA and then determined certain types of non-homogeneous restrictions that do not result in the observed error. In the Podinovski (2004b) model, there are no lower bounds on the input weights and no upper bounds on the output weights to correctly measure the efficiency of the DMUs. Sarrico and Dyson (2004) incorporated a virtual weight restriction and a virtual AR instead of the absolute weight restrictions into the DEA model in order to supply a natural representation of the decision makers' preferences. They also showed that *proportional* weight restrictions can lead to infeasible solutions in DEA problems.

Bernroider and Stix (2007) proposed a DEA method using weight restrictions to provide more significant information on the stability and validity of the results. Estellita Lins, Moreira da Silva, and Lovell (2007) addressed the problem of infeasibility of the developed weight restrictions in LPs. They proposed a theorem to demonstrate the feasibility conditions for the DEA multiplier programs with weight restrictions. Cooper, Ruiz, and Sirvent (2007) proposed a two-step procedure for selecting the weights in conjunction with the efficient facets of the highest possible dimension of the frontier in the DEA multiplier model. They showed that optimal solutions of the multiplier DEA formulation have alternate optima for the weights. Kuosmanen, Cherchye, and Sipiläinen (2006) adopted the so-called Law of One Price to the DEA model with weight restrictions. Their method was able to handle firm-specific output weights and variable returns to scale along with maintaining the linearity of the original DEA model. Liu and Peng (2008) introduced a DEA method to find the most favorable CSW to discern the difference between the efficient DMUs in view of maximizing the group's efficiency score.

Wang, Chin, and Poon (2008) proposed the DEA-AR model for weight derivation in the analytic hierarchy process to overcome the shortcomings of illogical local weights, over-insensitivity to some comparisons, information loss, and the overestimation of some local weights. Meng, Zhang, Qi, and Liu (2008) developed a two-level nonlinear frontier model where inputs and outputs with similar characteristics are aggregated into input and output groups, respectively, in order to increase the discrimination power. The weights among different classes were obtained using the conventional DEA models while the weights within groups were identified by a weighted-average DEA method. Kao (2008) modified the model presented by Meng et al. (2008) by converting the nonlinear model into a linear model using a variable substitution method. Zhiani Rezai and Davoodi (2011) showed that the cone-ratio weight restriction method developed by Cooper, Seiford, and Tone (1999) is a general case of the two-level DEA model studied by Meng et al. (2008) and Kao (2008). Wang, Luo, and Liang (2009) proposed an alternative DEA ranking method by imposing the minimum weight restrictions on inputs and outputs when the factor weights are determined through a set of maximin problems. Liu and Peng (2009) developed a systematic procedure to search the CSW for preferable and robust rankings by using the virtual weights restriction.

Wu, Liang, and Yang (2009a) proposed a cross-efficiency evaluation method to assess the performance of the nations participating in the Olympic games, similar to an AR application in Li, Liang, Chen, and Morita (2008). They incorporated the weight restrictions into their model to assure ordinal valuations among the obtained medals as disaggregated outputs. Khalili, Camanho, Portela, and Alirezaee (2010) adjusted the ARII model by introducing a nonlinear model that overcomes the shortcomings of the conventional ARII involving underestimation of efficiency and