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Group decision making with fuzzy linguistic preference relations via cooperative games method

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ABSTRACT

When we consider the weighting approach for group decision making with fuzzy linguistic preference relations, the groupment of experts has merely been studied. In this paper, a novel weighting approach on the basis of cooperative games method is developed. The group decision error matrix is built to reflect the deviations of all experts with given initial weighting vector. An iterative algorithm is designed to lower the sum of the decision error so that a final convergence result can be obtained. The advantage of the weighting algorithm is that it can consider the contribution of each expert and reduce the sum of decision error with increasing iteration numbers. Then an optimization model using triangular fuzzy numbers as alternatives' weights is constructed, whose results are used to rank the alternatives. Finally, a numerical example of subjective evaluation of vehicle sound quality is considered to illustrate the feasibility and validity of the proposed weighting approach in the group decision making problem.

1. Introduction

Group decision making (GDM) approaches have been widely studied in recent decades for producing a decision from a set of individuals. Usually, there are two phrases: (1) Aggregation phrase: to each alternative, a collective preference value is combined according to individual preferences by using certain aggregation tools. (2) Exploitation phase: in this phrase, the collective preference values are utilized to select the best alternative(s). For the reason that in practice the preference relation for any two alternatives given by an individual is the perception obtained from an appropriate semantic scale, the pairwise comparison values are provided with qualitative description rather than numerical values (Zadeh, 1975), i.e., the decision makes usually provide the fuzzy linguistic preference relations (Fuzzy LinPreRa) in the GDM, which has been widely applied to many real decision problems (Herrera, Herrera-Viedma, & Verdegay, 1995; Bordogna, Fedrizzi, & Pasi, 1997; Dong, Xu, & Li, 2008; citebib16; Shen, Olfat, Govindan, Khodaverdi, & Diabat, 2011; Xu, Ma, Tao, & Wang, 2013; Chen, Lin, & Lee, 2014; Chen et al., 2014). During the aggregation phrase, the weighting method is of great importance, because the weighting vector produced by the certain weighting method would affect the final aggregating results directly. At present, many weighting methods have been developed to derive the weighting vector in the GDM, such as the optimization models (Zhou, Chen, & Han, 2011; Zhou & Chen, 2013; Xu & Wu, 2013), straightforward construction methods (Xu, 2008), the entropy weight (Zamri & Abdullah, 2013) and the linguistic quantifiers' method (Yager, 1988; Dong, Xu, & Yu, 2009; Tapia García, del Moral, Martínez, & Herrera-Viedma, 2012).

Recently, Belenky (2002) analyzed a particular class of games of choosing partners and forming coalitions. Chen and Larbani (2006) derived the weights of a multi-attribute decision making with a fuzzy decision matrix by formulating it as a two-person zero-sum game with an uncertain payoff matrix. AL-Mutairi (2010) proposed a decision making with two decision makers by applying the cooperative games, whose decision information is fuzzy preference. Yu, Xu, and Chen (2011)constructed a multi-attribute aggregation process based on game theory. Madani and Lund (2011) suggested modeling multi-criteria decision making problems as strategic games and solving them by using non-cooperative game theory concepts. Sun et al. (2012) put forward a framework based on cooperative game theory to evaluate the power of the feature and then developed a general filter the feature selection scheme. Al-Dhanhani, Mizouni, Otrok, and Al-Rubaie (2014) provided a

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model to handle the free riding behavior in educational social application using game theory. Deng et al. (2014) proposed an evidential game theory framework to address the multi-criteria decision making process in the competitive environment. These applications to game models give a new perspective to study decision making problems.

Although types of game models have been applied to multi-attribute decision making, few works have paid attention to the groupment of the GDM. i.e., how to determine the importance of each decision maker in the GDM is an interesting issue, which has not been reported up to now. In fact, a GDM problem can be studied from the perspective of cooperative game model. Decision makers are regarded as the players of cooperative game, and the negative value of the decision errors is considered as the total payoff of cooperative game in the process of group decision making. The cooperative game was first introduced by Neumann and Morgenstern (1953). Since then, it has been widely studied and applied to social and economic problems, such as the reliability theory (Szeto, 2011), decision analysis (Lozano, 2012), agricultural marketing (Agbo, Rousselière, & Salanié, 2014), fuzzy set theory (Mallozzi, Scalzo, & Tijs, 2011; Borkotokey, Hazarika, & Mesiar, 2015) and location problem (Puerto, Tamir, & Perea, 2011, 2012). The Shapley value (Shapley, 1953) provides an objective way to determine how important each player is, and how much payoff he/she can reasonably expect in the cooperative game. The Shapley value method is a general and useful tool to deal with the two characteristics. Luce and Raiffa (1957), Owen (1995), Kelly (2003) and AL-Mutairi (2010) have discussed the Shapley value in games model. However, such method has rarely been applied to decision making problems.

In this paper, the Shapley value method is mainly used to derive the weighting vector of group decision making with fuzzy linguistic preference relations. To do this, the rest of the paper is as follows. In Section 2, some basic concepts and operations with Fuzzy LinPreRa and the notions of cooperative games and Shapley function are reviewed. Section 3 devotes to propose a novel weighting method for GDM with Fuzzy LinPreRa using cooperative games. In Section 4, a numerical example is developed to illustrate the feasibility and validity of the proposed method. Some conclusions and possible future works are summarized in Section 5.

2. Preliminaries

In this section, some basic notations and operations involved in the theory of cooperative games method are presented for further consideration.

2.1. Fuzzy LinPreRa approach

Given that $\boldsymbol{S} = \{s_0, s_1, \ldots, s_g\}$ (g is an even number) is a linguistic term set, let $\boldsymbol{X} = \{x_1, x_2, \ldots, x_n\}$ be a set of alternatives, a fuzzy linguistic preference relation (Fuzzy LinPreRa, for short) $\tilde{P} = (\tilde{p}_{ij})_{n\times n} = (p_{ij}^L, p_{ij}^M, p_{ij}^R)_{n\times n}$ on X is a fuzzy linguistic assessment set on the product set $\boldsymbol{X} \times \boldsymbol{X}$, i.e., the fuzzy linguistic assessment variables (Table 1) $\tilde{p}_{ij} = (p_{ij}^L, p_{ij}^M, p_{ij}^R)$ are provided to replace crisp values and represent corresponding fuzzy linguistic preference of x_i with respect to x_j.

Table 1
Fuzzy linguistic assessment variables.

Linguistic variables	Fuzzy numbers
Perfect (P)	$(p_P^L, p_P^M, 1)$
⋮	⋮
Medium (M)	$(p_M^L, 0.5, p_M^R)$
⋮	⋮
None (N)	$(0, p_N^M, p_N^R)$

The advantage of this approach is that it allows individual to provide vague or imprecise opinion when he/she is in the process of comparing any two alternatives.

The following notions and operations are considered for further studying:

Definition 2.1 (Wang and Chen, 2005). Let $\tilde{P} = (\tilde{p}_{ij})_{n\times n} = (p_{ij}^L, p_{ij}^M, p_{ij}^R)_{n\times n}$ be a Fuzzy LinPreRa. $\tilde{P}$ is said to be fuzzy linguistic complementary judgment matrix if the following properties hold:

(1) $p_{ii}^L = 0.5,\ p_{ii}^M = 0.5,\ p_{ii}^R = 0.5,\ i \in \{1, 2, \ldots, n\}$,
(2) $p_{ij}^L + p_{ji}^R = p_{ij}^M + p_{ji}^M = p_{ij}^R + p_{ji}^L = 1,\ i, j \in \{1, 2, \ldots, n\},\ i \neq j$.

The consistency of fuzzy linguistic complementary judgment matrix $\tilde{P}$ is the assurance of right decision, which can be defined as follows:

Definition 2.2 (Wang and Chen, 2005). $\tilde{P} = (p_{ij}^L, p_{ij}^M, p_{ij}^R)_{n\times n}$ is said to be consistent if the following statements hold: $p_{ij}^L + p_{jk}^L + p_{ki}^R = \frac{3}{2}$, $p_{ij}^M + p_{jk}^M + p_{ki}^M = \frac{3}{2}$ and $p_{ij}^R + p_{jk}^R + p_{ki}^L = \frac{3}{2}$ for any $i < j < k$.

The consistency test and modification approach of fuzzy linguistic complementary judgment matrix have been widely studied (Dong et al., 2008; Tapia García et al., 2012; Dong, Hong, & Li, 2013; Xu & Wu, 2013; Zhang, Dong, & Xu, 2014). Hereinafter, the modification approach proposed by Xu and Wang (2013) will be used to obtain the consistent Fuzzy LinPreRa matrix of each decision maker.

When we make a final decision, the operations of fuzzy linguistic variables are useful tools (Chen & Hwang, 1993). For any fuzzy numbers $\tilde{A}_1 = (l_1, m_1, r_1)$, $\tilde{A}_2 = (l_2, m_2, r_2)$ and $\lambda \geqslant 0$, then.

(1) Addition: $\tilde{A}_1 \oplus \tilde{A}_2 = (l_1 + l_2, m_1 + m_2, r_1 + r_2)$.
(2) Subtraction: $\tilde{A}_1 - \tilde{A}_2 = (l_1 - r_2, m_1 - m_2, r_1 - l_2)$.
(3) Scalar-multiplication: $\lambda\tilde{A}_1 = (\lambda l_1, \lambda m_1, \lambda r_1)$.

The defuzzified value of a triangular fuzzy number is defined as follows:

Definition 2.3 (Bortolan and Degani, 1985). Let $\tilde{A} = (l, m, r)$ be a triangular fuzzy number, then the following formula

$$a = \frac{l + m + n}{3} \tag{1}$$

is said to be the defuzzified value of $\tilde{A}$.

2.2. Cooperative games method

Neumann and Morgenstern (1953) developed the basic notions of cooperative game theory and the elementary properties. Since then, lots of works have been developed to analyze kinds of cooperative games (Luce & Raiffa, 1957; Owen, 1995; Belenky, 2002; Kelly, 2003; AL-Mutairi, 2010; Sun et al., 2012).

Definition 2.4 (*Neumann and Morgenstern, 1953; Neumann and der Gesellschaftsspiele, 1959*). A cooperative game is given by specifying a value for every coalition. Formally, the game consists of a finite set of N players, called the grand coalition, and a characteristic function $v : 2^N \to \mathbb{R}$ from the set of all possible coalitions of players to a set of payments that satisfies $v(\varnothing) = 0$.