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An efficient hybrid technique for numerical optimization and applications ☆

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ABSTRACT

Real World Optimization Problems is one of the major concerns to show the potential and effectiveness of an optimization algorithm. In this context, a hybrid algorithm of two popular heuristics namely Differential Evolution (DE) and Particle Swarm Optimization (PSO) engaged on a 'tri-population' environment. Initially, the whole population (in increasing order of fitness) is divided into three groups – Inferior Group, Mid Group and Superior Group. DE is employed in the inferior and superior groups, whereas PSO is used in the mid-group. The proposed method is abbreviated as DPD as it uses DE–PSO–DE on a population. Two strategies namely *Elitism* (to retain the best obtained values so far) and *Non-redundant search* (to improve the solution quality) have been additionally employed in DPD cycle. Moreover, the robustness of the mutation strategies of DE have been well studied and suitable mutation strategies for both DEs (for DPD) are investigated over a set of existing 8 popular mutation strategies which results 64 variants of DPD. The top DPD is further tested through the test functions of CEC2006, CEC2010 and 5 Engineering Design Problems. Also it is used to solve CEC2011 Real World Optimization problems. An excellent efficiency of the recommended DPD is confirmed over the state-of-the-art algorithms.

1. Introduction

Many Real World Problems involve inequality and/or equality constraints and are thus posed as constrained optimization problems. These problems are highly nonlinear, convex or non-convex and smooth or non-smooth along with a large number of design variables and constraints, often faced with multiple local optima. Modern evolutionary optimization techniques appear to be more efficient in solving constrained optimization problems because of their ability to seek the global optimal solution. Among them, PSO (Kennedy & Eberhart, 1995) and DE (Storn & Price, 1997) are recognized as the most promising methods for solving a wide range of problems. These optimization techniques do not always guarantee discovering the globally optimal solution in a finite time, they often provide a fast and reasonable solution (sub-optimal, nearly global optimal). Also certain shortcomings associated with them which sometimes deteriorate the performance of the algorithms. Unfortunately, according to 'No Free Lunch Theorem (Wolpert & Macready, 1997)', no single optimization method exist which is able to solve all global optimization problems, consistently. Therefore number of attempts to solve optimization problems, while hybrid algorithms have shown outstanding reliability and efficiency to solve these problems. In fact, the hybrid techniques, being powerful, yields promising results in solving specific problems.

In the past decade, many hybrids of DE and PSO have been proposed in the recent literature. Some of the year wise developments on the DE–PSO hybridization are discussed below.

Author (year)	Technique used	Application
Hendtlass (2001)	SDEA	Unconstrained global optimization
Zhang and Xie (2003)	DEPSO	Unconstrained global optimization
Talbi and Batouche (2004)	DEPSO	Medical image processing
Das, Konar, and Chakraborty (2005)	PSO-DV	Unconstrained global optimization
Moore and Venayagamoorthy (2006)	DEPSO-MV	Multi-objective optimization

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Author (year)	Technique used	Application
Hao, Guo, and Huang (2007)	DEPSO	Unconstrained global optimization
Omran, Engelbrecht, and Salman (2008)	BBDE	Unconstrained optimization problems and image processing
Zhang, Ning, Lu, Ouyang, and Ding (2009)	DE–PSO	Unconstrained global optimization
Khamsawang, Wannakarn, and Jiriwibhakorn (2010)	PSO–DE	Power systems
Lu, Sriyanyong, Song, and Dillon (2010)	CBPSO-RVM	Unconstrained and Power systems optimization
Liu, Cai, and Wang (2010)	PSO–DE	Constrained and engineering optimization
Niknam, Mojarrad, and Meymand (2011)	FAPSO-VDE	Power systems
Pant and Thangaraj (2011)	DE–PSO	Unconstrained and real life problems optimization
El Dor, Clerc, and Siarry (2012a)	DEPSO-2S	Unconstrained Real life problems
Satapathy et al. (2012)	IPSODETEA	Data Clustering
Nwankwor, Nagar, and Reid (2012)	HPSDE	Well placement optimization
Araújo and Uturbey (2013)	PSO–DE	Power systems
Sayah and Hamouda (2013)	DEPSO	Power systems

Clearly, in the process of hybridization of DE and PSO one method takes the advantage of the other. This fact is reported in the following papers and their proposed methods are applied in many ways as given below.

Author (year)	Technique used	Application
Das, Abraham, and Konar (2008)	PSO-DV	Engineering design
Thangaraj, Pant, Abraham, and Bouvry (2011)	DE–PSO, AMPSO, GA-PSO	Unconstrained global optimization
Xin, Chen, Zhang, Fang, and Peng (2012)	–	Review and Taxonomy of hybrid DE and PSO

However, there has been a continuous modification in the operators and/or the way of applying them. In this study, the simultaneous use of DE and PSO on different part of the same population is referred to '**parallel**' whereas their alternate use on entire population with respect to the generations, without breaking the population to parts is referred to '**sequential**'. In recent years, parallel employment of DE and PSO is preferred over the sequential one. Few of such parallel usage available in the literature are briefly presented below.

Author (year)	No. of breakups in a population	Proposed Technique	Application
Cagnina, Esquivel, and Coello Coello (2007)	2	CPSO-shake	Constrained optimization
Wang, Yang, and Zhao (2010)	3	DEDEPSO	Unconstrained global optimization
Cagnina, Esquivel, and Coello Coello (2011)	2	CPSO-shake	Constrained optimization
Elsayed, Sarker, and Essam (2011)	4	SAMO-GA, SAMO-DE	Constrained optimization
Han, Liao, Chang, and Lin (2013)	2	GDE	Unconstrained global optimization
Zhang, Cheng, Gheorghe, and Meng (2013)	5	DETPS	Constrained optimization
Yadav and Deep (2014)	2	CSHPSO	Constrained optimization and Power systems
Kordestani, Rezvanian, and Meybodi (2014)	2	CDEPSO	Dynamic optimization problems

Based on the earlier works and inspired by the recent works on population-breakup concept, a further study is being carried out in this paper to improve the robustness of the hybridization of DE and PSO in a different fashion. In Cagnina et al. (2007, 2011), the higher break up of population is not encouraged due to the failure of neighborhood topology. However, the tri-breakup is well preferred over the bi-breakup (Branke, 1999; Wang, Wang, & Yang, 2007; Wang et al., 2010). Hence, the 'tri-break up of population' is chosen in this present study depending on the considered population size in the paper and henceforth is recalled as 'tri-population'. The novel hybrid algorithm thus proposed is named as DE–PSO–DE (DPD) for solving Real World Optimization problems.

The reminder of this paper is organized as follows: Section 2 presents brief review of DE and PSO. Section 3 presents constraint handling technique used in this present study. The proposed algorithm is described along with the selection of best suit mutation operators for DEs employed in the proposed algorithm, in Section 4. Section 4.2 includes comparison of DPD with latest existing algorithms. Implementation of DPD for CEC'11 Real World Optimization problems presented in Section 5. Finally, the conclusion is drawn in Section 6.

2. Brief on DE and PSO

2.1. Differential Evolution (DE)

DE starts with a random population of a fixed number of D-dimensional search variable vectors. DE works with mutation, crossover and selection; which are explained briefly as follow.

Mutation: Mutation is a vital operator in DE. In a particular generation, for each *i*th vector $X_i = (x_{i1}, x_{i2}, \ldots, x_{iD})$ in the