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Uncertainty index based consistency measurement and priority generation with interval probabilities in the analytic hierarchy process ☆

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A R T I C L E I N F O

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A B S T R A C T

The pairwise judgments provided by a decision-maker are often inconsistent in the analytic hierarchy process. Dealing with inconsistency among the given judgments by uncertainty and interval probabilities has received more attention in recent years. In this paper, an interval multiplicative reciprocal matrix is constructed to capture inconsistency and the decision-maker's judgments from a multiplicative reciprocal comparison matrix (MRCM). A geometric mean based uncertainty index is defined to measure the uncertainty level of the constructed interval matrix, and used to check acceptable consistency of MRCMs. The paper devises a parameterization approximate relation between the normalized interval probabilities and the constructed interval matrix. A two-stage procedure consisting of two optimization models is developed to generate interval probabilities from MRCMs. The first stage minimizes the power-sum of absolute deviations between the logarithms of the sides of the approximate relation. The second stage aims to seek the most suitable interval probability vector among the optimal solutions derived from the previous stage such that inconsistency of the original MRCM is reflected by interval probabilities as much as possible. An interval probability based algorithm is further put forward for solving multi-criteria decision making problems with MRCMs. Two numerical examples and comparison analyses are presented to demonstrate the performance and validity of the proposed models.

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1. Introduction

The analytic hierarchy process (AHP) (Saaty, 1980) is a popular and useful tool for multi-criteria decision making (MCDM) analysis (Durbach & Stewart, 2012), and has been widely applied in practice (Ho, 2008). An excellent summary of main developments in AHP was given in Ishizaka and Labib (2011). The contemporaneous applications of AHP involve in supply chain management (Bahinipati, Kanda, & Deshmukh, 2009; Byrne, Heavey, Blake, & Liston, 2013; Chen & Wu, 2013; Cho, Lee, Ahn, & Hwang, 2012; Ruiz-Torres, Mahmoodi, & Zeng, 2013), risk evaluation (Song, Ming, & Xu, 2013; Wang, Liu, & Elhag, 2008), strategy selection (Chang, Wu, Lin, & Chen, 2007; Chen & Wang, 2010), weapon system selection (Lee, Kang, Rosenberger, & Kim, 2010), project evaluation and selection (Mohajeri & Amin, 2010; Vidal, Marle, & Bocquet, 2011), to name a few.

In AHP, pairwise comparison ratios over decision objects that may be alternatives or criteria are furnished by a decision-maker, and structured as a multiplicative reciprocal comparison matrix (MRCM). As the decision-maker's judgment is subjective and an object is compared with others many times, the pairwise judgments are often inconsistent, especially in decision situations involving a large number of objects. Saaty (1980) defined multiplicative consistency and suggested that the application of AHP allows a certain inconsistency level for MRCMs. He introduced the consistency index (CI) to measure the inconsistency level of the decision-maker's judgments by employing the largest eigenvalue of the comparison matrix, and proposed the notion of acceptable consistency by the consistency ratio (CR) checking method. Aguaron and Moreno-Jimenez (2003) formalized the inconsistency measure equation given by Crawford and Williams (1985) and put forward the geometric consistency index (GCI). They also developed the GCI threshold that is analogous to Saaty's CR. Recently, inconsistency of MRCMs is viewed as a certain kind of uncertainty in the given pairwise judgments, and interval probabilities are derived from MRCMs and used to reflect such uncertainty (Entani, 2013; Entani & Tanaka, 2007; Entani & Sugihara, 2012; Guo & Tanaka, 2010; Guo & Wang, 2012). A major challenge is the capture and checking of the uncertainty among pairwise judgments, which leads our motivation to study the uncertainty index based consistency measurement for MRCMs.

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Another important research topic in AHP is the elicitation of priorities from MRCMs. A number of prioritization methods have been developed to derive crisp priorities, such as the eigenvector (EV) method (Saaty, 1980), the row geometric mean (RGM) (or called the logarithmic least square (LLS)) method (Crawford & Williams, 1985), the logarithmic lease absolute value (LLAV) method (Cook & Kress, 1988), the direct least square (DLS) method and the weighted least square (WLS) method (Chu, Kalaba, & Springam, 1979), the logarithmic absolute error (LAE) method (Budescu, 1984) and goal programming methods (Bryson, 1995; Grzybowski, 2010; Lam & Choo, 1995; Lin, 2006). Among these methods perhaps EV and RGM are the most generally adopted in AHP. It is difficult to identify which method performs better due to the fact that each method has its advantages and disadvantages. It is noted that these methods yield the same crisp priority vector and ranking of objects when a MRCM is multiplicatively consistent. However, if a MRCM is inconsistent, different priority vectors are generated by these different methods and objects may be ranked as diverse orders. This result reveals that there exists uncertainty of preference intensities and rankings of objects when object priorities are elicited from inconsistent MRCMs. Therefore, it is more logical and reasonable to elicit interval probabilities of priorities from inconsistent MRCMs. This leads to the second motivation of the paper, which is to develop some optimization models for deriving interval probabilities from MRCMs.

The paper first constructs an interval multiplicative reciprocal matrix (Saaty & Vargas, 1987) to capture the pairwise judgments and inconsistency in the original MRCM. A geometric mean based uncertainty index is then defined to measure the uncertainty level of the constructed interval multiplicative reciprocal matrix. It is shown that this uncertainty index can be equivalently expressed as GCI proposed by Aguaron and Moreno-Jimenez (2003), and employed to check acceptable consistency of MRCMs. Subsequently, a parameterized relation is devised to approximate interval ratios in the constructed interval matrix by the normalized interval probabilities. A two-stage procedure consisting of two optimization models is established to obtain interval probabilities from MRCMs. The first optimization model is developed to minimize the power-sum of absolute deviations between the logarithms of the sides of the approximate relation such that the constructed interval matrix is approximated as much as possible. By incorporating the optimal objective value of the first optimization model into its constraints, the second optimization model is devised to find the most suitable interval probability vector such that the uncertainty of the constructed matrix is reflected by interval probabilities as much as possible. Finally, an interval probability based algorithm is developed to solve MCDM problems with MRCMs by combining these models together.

The paper is organized as follows. Section 2 introduces main concepts and results relating to multiplicatively consistent MRCMs, Saaty's CI and the approximated GCI thresholds. Section 3 shows how to measure inconsistency level of MRCMs from the viewpoint of uncertainty. In Section 4, some optimization models are developed to generate interval probabilities from MRCMs and an algorithm is proposed to solve MCDM problems with MRCMs. Section 5 provides two illustrative examples and comparison analyses. The main conclusions are given in Section 6.

2. Preliminaries

Consider a MCDM problem with a set of objects $X = \{x_1, x_2, \ldots, x_n\}$, where the objects could be either alternatives or criteria. A decision-maker employs the pairwise comparison technique to elicit his/her judgments over X. In the AHP method, these judgments are structured by a MRCM $A = (a_{ij})_{n\times n}$, where a_{ij} gives a ratio-based preference intensity of the object x_i over x_j such that

$$a_{ij} > 0, \quad a_{ij}a_{ji} = 1, \quad a_{ii} = 1 \quad \text{for all} \quad i,j = 1,2,\ldots,n \tag{2.1}$$

If $a_{ij} > 1$, then the object x_i is preferred to x_j and the larger the a_{ij}, the stronger the preferred intensity is. If $a_{ij} = 1$, then the objects x_i and x_j are equally preferred. If $a_{ij} < 1$, then the object x_i is not preferred to x_j and the smaller the a_{ij}, the stronger the object x_j is superior to x_i.

The consistency of MRCMs has been investigated by many researchers (Hartvigsen, 2005; Jensen & Hicks, 1993; Li & Ma, 2007; Saaty, 1980). Saaty (1980) introduced a multiplicative transitivity equation to define consistency of MRCMs as follows.

Definition 2.1 Saaty (1980). Let $A = (a_{ij})_{n\times n}$ be a MRCM. A is multiplicatively consistent, if it satisfies the following transitivity equation:

$$a_{ij} = a_{ik}a_{kj} \quad \text{for all} \quad i,j,k = 1,2,\ldots,n \tag{2.2}$$

If A is multiplicatively consistent, then the decision-maker's judgments are consistent and can be expressed as $a_{ij} = \omega_i/\omega_j$ (i,j = 1, 2, ..., n), where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the normalized crisp priority weight vector derived from A.

If A is not a multiplicatively consistent MRCM, then the decision-maker's judgments are inconsistent and there exist differences between a_{ij} and ω_i/ω_j for some i,j = 1, 2, ..., n. In this case, the derived crisp priority weight ω_i (i = 1, 2, ..., n) approximates the preference intensity of x_i.

Saaty (1980) proposed the EV method to derive a normalized crisp priority weight vector from a MRCM A, and introduced the following CI to measure the inconsistency level of the decision-maker's judgments in A.

$$CI = \frac{\lambda_{\max} - n}{n-1} \tag{2.3}$$

where $\lambda_{\max}$ is the largest eigenvalue of the eigenvector problem $A\omega = \lambda\omega$, and n is the order of A.

Aguaron and Moreno-Jimenez (2003) stated that Saaty's CI can be formulated as:

$$CI = \frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{j=1, j\neq i}^{n}(a_{ij}\omega_j/\omega_i - 1) \tag{2.4}$$

where $(\omega_1, \omega_2, \ldots, \omega_n)^T$ is the priority vector derived from A by the EV method.

It can be seen from (2.4) that CI measure does not take into account the multiplicative reciprocal property of the difference ratio between a_{ij} and ω_i/ω_j.

If A is multiplicatively consistent, then $\lambda_{\max} = n$, implying $CI = 0$. However, for inconsistent MRCMs, we have $CI > 0$.

To determine a normalized consistency measure that is independent of the orders of MRCMs, Saaty proposed CR as follows.

$$CR = \frac{CI}{RI(n)} \tag{2.5}$$

where $RI(n)$ denotes the average CI of a large number of randomly generated MRCMs.

Saaty (1980) suggested that an acceptable threshold for CR is less than or equal to 0.1. In other words, if $CR \leqslant 0.1$, the MRCM A is said to be acceptably consistent. In this case, the derived priority weight ω_i (i = 1, 2, ..., n) by the EV method can adequately approximate the preference intensity of x_i. If $CR > 0.1$, A is said to be unacceptable and the decision-maker's judgments should be adjusted to improve the consistency level.

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