Computers & Industrial Engineering 90 (2015) 305–313

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

A hybrid differential evolution algorithm for multiple container loading problem with heterogeneous containers

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ARTICLE INFO

Article history:
Received 13 January 2015
Received in revised form 12 October 2015
Accepted 13 October 2015
Available online 17 October 2015

Keywords:
Three-dimensional bin packing (3D-BPP)
Differential evolution (DE)
Mathematical programming
Evolutionary algorithms (EAs)

ABSTRACT

We consider a multiple container loading problem, commonly known as the three-dimensional bin packing problem (3D-BPP), which deals with maximizing container space utilization while the containers available for packing are heterogeneous, i.e., varying in size. The problem has wide applications in cargo transportation, warehouse management, medical packaging, and so on. We develop a differential evolution (DE) algorithm hybridized with a novel packing heuristic strategy, best-match-first (BMF), which generates a compact packing solution based on a given box packing sequence and a container loading sequence. The effectiveness of the proposed algorithm is evaluated on a set of industrial instances and randomly generated instances. The results show that the proposed algorithm outperforms existing solution approaches in terms of solution quality.

1. Introduction

The multiple container loading problem can be stated as follows. There are m total boxes and each box i has dimensions (l_i, w_i, h_i). These boxes need to be packed into N containers with various dimensions (L_j, W_j, H_j). The boxes can be rotated in six ways. The problem is to orthogonally pack all boxes into a subset of containers such that the space utilization ratio is maximized; or equivalently, the wasted space of containers is minimized. The problem we consider is a generalization of the classic three-dimensional bin packing problem (3D-BPP) in which containers are identical. For clarity, we denote the problem under study to 3D-BPP-*h*, where *h* stands for heterogeneous, and we use the notation of 3D-BPP-*i* for the case of identical containers. The 3D-BPP-*i* is a well known operations research optimization problem that can be used to model many industrial operations, i.e., container and pallet loading, cargo and warehouse management, medical supplies packaging, and so on. It has been reasonably well studied (Martello, Pisinger, & Vigo, 2000; Lodi, Martello, & Vigo, 2002; Lodi, Martello, & Vigo, 2004; Martello, Pisinger, Vigo, Boef, & Korst, 2007; Fekete, Schepers, & van der Veen, 2007; Gonçalves & Resende, 2013). A closely related problem is to pack boxes into a single container. A number of studies have focused on this problem, and both exact and heuristic algorithms have been proposed, e.g., (Martello et al., 2000; Padberg, 2000; Karabulut & İnceoğlu, 2005; Parreño, Alvarez-Valdes, Tamarit, & Oliveira, 2008; Kang, Moon, & Wang, 2012). However, in many industrial situations, containers are different in sizes. For example, in the shipping and transportation industry, several types of standard containers with different dimensions are used. For marine transport, standard containers of 20 or 40 feet length are used. For land or air transport, many kinds of containers are used (Takahara & Miyamoto, 2005).

As a NP-hard problem, 3D-BPP-*i* problems have attracted much interest in the operations research community, and both exact and heuristic algorithms for this problem have been developed in past decades. While exact algorithms can find optimal solutions, it usually takes huge amount of time to solve even just moderate-sized instances. Heuristic algorithms, which cannot guarantee optimality, are often capable of providing fairly effective solutions with much less computational effort. Meta-heuristic algorithms, i.e., genetic algorithm (GA), simulated annealing (SA), ant colony optimization (ACO), hold a reputation in solving difficult combinatorial optimization problems. The problem we consider, i.e., 3D-BPP-*h*, is NP-hard in the strong sense since it generalizes the 3D-BPP-*i*. In addition, it is more difficult in sense of combinatorial optimization because it not only needs to determine which boxes are packed together, but also what specific container they are packed in. Therefore, heuristics become the clear choice for solving real world industrial problems. In this paper, we present a heuristic method named differential evolution (DE) to solve 3D-BPP-*h*. Computational experiments are carried out on extensive random instances and some industrial instances. To the best of our knowledge, this is the first in the literature to tackle the 3D-BPP-*h* problem using the DE.

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http://dx.doi.org/10.1016/j.cie.2015.10.007
0360-8352/

The remainder of this paper is organized as follows. Section 2 reviews related literature. A mathematical formulation for the problem is presented in Section 3. Section 4 introduces the new approach with a detailed description of the DE and a novel packing strategy. Computational results are reported in Section 5. We conclude the study in Section 6.

2. Literature review

In the standard 3D-BPP-*i*, the rectangular boxes need to be packed orthogonally into a minimal number of rectangular containers of identical size. Martello et al. (2000) present exact and approximation algorithms for the 3D-BPP-*i* and discuss the lower bounds. Their algorithms are based on a concept called "corner points" which is used to track the feasible placement regions and a Branch & Bound procedure is employed to verify whether a set of boxes can be placed into a container. The algorithms are improved by Martello et al. (2007) with a new version of the procedure to compute corner points and an updated Branch & Bound algorithm. The algorithms are able to solve moderately large instances to optimality for the general 3D-BPP and its robot-packable variant. Fekete et al. (2007) develop a two-level tree search algorithm for solving higher-dimensional packing problems to optimality. Faroe, Pisinger, and Zachariasen (2003) provide a heuristic based on a guided local search for packing items with fixed orientations into a minimum number of identical bins. Lodi et al. (2002) develop a tabu search framework by exploiting a new constructive procedure for the problem where the carton orientations are fixed, i.e., they cannot be rotated, and a unified tabu search code for general multi-dimensional bin packing problems is developed later Lodi et al. (2004). Recently, Gonçalves and Resende (2013) present a biased random-key genetic algorithm (BRK-GA) for 2D and 3D bin packing problems in which a novel placement heuristic is proposed and hybridized in a genetic algorithm based on random keys. They use a concept of empty maximal-spaces to manage feasible placement positions, which is used in the heuristic to determine a bin and the free maximal space after each box is placed. Almeida and Figueiredo (2010) study the problem by considering some additional restrictions on the placement and propose a mathematical formulation as well as two special designed heuristic algorithms.

The 3D-BPP-*i* with a single bin aims to select a subset of boxes which can be packed into a container to pursue high utilization ratio. This problem is also called the single container loading problem (SCLP) in the literature. Padberg (2000) proposes an MILP formulation with a polyhedral analysis. Martello et al. (2000) present a two-level Branch & Bound algorithm. Their computation experiments demonstrate the algorithm can solve instances with 90 boxes to optimality in a few minutes. Kang et al. (2012) present a hybrid genetic algorithm for a 3D-BPP in which boxes are packed into a single bin to maximize the number of boxes packed. They propose a heuristic packing strategy referred to as the Improved-Deepest-Bottom-Left with Fill (I-DBLF) algorithm based on the deepest bottom left packing method developed by Karabulut and İnceoğlu (2005). Computational efficiency of the proposed packing strategy is achieved by introducing cuboid space objects which allow to pre-check the feasibility of box insertion. Liu, Tan, Xu, and Liu (2014) present a binary tree search algorithm for the SCLP. In this algorithm, all the boxes are grouped into strips and layers and generate solutions that satisfy the full support constraint, orientation constraint and guillotine cutting constraint. Zheng, Chien, and Gen (2015) develop a multi-objective genetic algorithm with a multi-population strategy and a fuzzy logic controller (FLC) to maximize the container space utilization and the value of total loaded boxes. Another variation of this problem is referred to as the open dimension problem (ODP). The ODP allows a single variable dimension to occur in the packing planning process. Bortfeldt and Mack (2007) present a layer-building heuristic method for the 3D-BPP which aims to packing boxes into a single container to minimize required container length. Wu, Li, Goh, and de Souza (2010) study a 3D-BPP problem with variable bin heights. They first propose an exact mathematical model based on Chen, Lee, and Shen (1995)'s model and then design a genetic algorithm to hybridize with a heuristic packing procedure. This problem is further studied by He, Wu, and de Souza (2012), and the authors develop an improved genetic algorithm for the 3D-BPP with variable carton orientations. They improve the decoding procedure for tighter packing and higher computational efficiency, then a novel global search framework (GSF) is proposed based on an evolutionary gradient method.

Unlike the 3D-BPP-*i*, little attention has been paid in the literature so far to the 3D-BPP-*h*. Chen et al. (1995) seem to be the first one providing a 0–1 mixed integer linear programming (MILP) model to solve the 3D-BPP-*h* with variable orientations and various bins sizes. The model uses left, right, top and bottom decision variables to determine the relative positions of boxes and orientation variables to mimic different packing orientations. However, this MILP model can only solve small instances to optimality. A very similar problem is studied by Takahara and Miyamoto (2005), in which an evolutionary approach using GA is proposed, where a pair of sequences (one for boxes and another for containers) is used as the genotype, and a heuristic is used to determine the loading plan given the sequence of boxes and containers. They use a package loading heuristic called "Branch Heuristics" to convert a chromosome to a packing solution. An obvious drawback of this heuristic procedure is that it fails to make full use of the available space. Eley (2003) proposes an approach based on a set partitioning formulation and use a tree search based heuristic to pre-generate single bin packing patterns. Their model is extended by Che, Huang, Lim, and Zhu (2011) and Zhu, Huang, and Lim (2012), who propose a heuristic and an approximation algorithm to generate columns. This line of methods may fail to solve problems when containers are heterogeneous since the patterns that need to be generated increase exponentially with the number of container types. Li, Zhao, and Zhang (2014) study a 3D-BPP with heterogeneous bins using a genetic algorithm. Recently, Alvarez-Valdes, Parreño, and Tamarit (2015) study the problem where several types of bins of different sizes and costs are available and the objective is to minimize the total cost of the bins used for packing the boxes. They develop a new lower bound for the problem based on an integer programming formulation.

In this paper, we study 3D-BPP-*h* with variable orientations and various bins sizes. A differential evolution algorithm with a heuristic procedure is proposed. In the algorithm, a chromosome contains a box packing sequence, and a container loading sequence is used to heuristically generate packing solution. These sequences evolve in the differential evolution algorithm which leads to improved solutions. The heuristic procedure is able to decode these sequences to a compact packing solution.

3. Mathematical programming model

We define the following notations. There are total m boxes indexed as 1, ..., m to pack, and total N containers indexed as 1, ..., N are available. These containers have different sizes. The length, width, and height of product i are denoted by l_i, w_i, and h_i, while the length, width, and height of box j are denoted by L_j, W_j, and H_j. The boxes can be rotated in six ways which are indexed by k (Fig. 1). The problem is to find an assignment of the boxes to the containers, i.e., placing the boxes in which container