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Economic and economic-statistical designs of the side sensitive group runs chart ☆

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ABSTRACT

The idea of proposing the economic and economic-statistical designs of the side sensitive group runs (SSGR) chart is presented in this paper. In the economic design, a simplified algorithm is used to search for the optimal design parameters that minimize the expected hourly cost. Nevertheless, this design has a major weakness, where it overlooks the statistical performance of the control chart. Therefore, in order to improve the effectiveness of the control chart in detecting process shifts, the economic-statistical design takes into account the statistical properties while the cost is minimized by placing statistical constraints upon the cost model of the economic design. Besides formulating the economic and economic-statistical designs based on the average run length (*ARL*), the economic and economic-statistical designs of the SSGR chart are also formulated based on the expected average run length (*EARL*) since the process shift size is usually unknown in real situations. In this paper, the sensitivity analyses of the optimal cost and the optimal design parameters are implemented for various input parameters. The effects of misspecification of the shift size on the performance of the SSGR chart are also illustrated based on numerical examples for different input parameters. This paper will also look at whether the SSGR chart performs economically better than the Shewhart $\overline{X}$, synthetic, group runs (GR) and EWMA charts in the economic-statistical design based on the *EARL*. From the results of comparison, it is shown that the economic performance of the SSGR chart is better than that of the other four control charts in most practical situations.

1. Introduction

Statistical Process Control (SPC) refers to a collection of statistical methods used extensively to monitor and improve the quality and productivity of industrial processes and service operations. A control chart is one of the most useful techniques in SPC. The Shewhart $\overline{X}$ chart has been deemed as the most important statistical tool in process surveillance. However, the main limitation of this chart is its poor statistical efficiency towards the detection of small and moderate process mean shifts. Therefore, more research works have been conducted to suggest increasingly effective tools for statistically monitoring the quality of products and processes. Recently, many researchers have contributed to a wide variety of control charts to improve process monitoring, such as Haq, Brown, Moltchanova, and Al-Omari (2015), Zhang, Tsung, and Zou (2015), Costa and Machado (2015), Rakitzis, Castagliola, and Maravelakis (2015), Lee, Park, and Jun (2014), Tuerhong and Kim (2014), Chong, Khoo, and Castagliola (2014), Bersimis, Koutras, and Maravelakis (2014), Haridy, Wu, Abhary, Castagliola, and Shamsuzzaman (2014), Teoh, Khoo, Castagliola, and Chakraboti (2014), and many more.

The performance criteria used in order to evaluate the effectiveness of the control chart can be based either on a statistical criterion or an economic criterion. The statistical performance of a control chart is implemented by considering the time required to detect process shifts, while the economic performance refers to the cost related to the control chart in process monitoring. As control charts became ubiquitous in industrial application, the significance of the economic impact has aroused interest among scholars to investigate the minimization of the cost by selecting the optimal design parameters of control charts.

The economic design was first proposed by Duncan (1956). Considerable attention has then been given to the economic design of other control charts after the pioneering work of Duncan. Lorenzen and Vance (1986) generalized Duncan's model with its unified approach. However, this cost model is more complicated to understand. Chung (1991) introduced a new simplified optimum solution to resolve the cost model of Lorenzen & Vance.

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Chung's algorithm was proven better in that it takes a more precise form and it is effective in computing the optimal design parameters and minimizing the cost model. Therefore, the optimal procedure of Chung is employed by other researchers to create a more efficient control chart in the economic design, including Xie, Tang, and Goh (2001) and Zhang, Xie, and Goh (2008). More recent works on economic approaches on a wide variety of control charts were made by Trovato, Castagliola, Celano, and Fichera (2010), Zhang, Xie, Goh, and Shamsuzzaman (2011), Pandey, Kulkarni, and Vrat (2011), Tsai, Chiang, and Chang (2011), Lupo (2014), Niaki, Gazaneh, and Toosheghanian (2013), Yilmaz and Burnak (2013), Bahiraee and Raissi (2014), Saghaei, Ghomi, and Jaberi (2014), Vommi and Kasarapu (2014), Zhang, Su, Li, and Wang (2014) and Guo, Cheng, and Lu (2014).

Although it is rational to illustrate the model of a control chart from an economic perspective, the economic design has its major weakness as it overlooks the statistical performance of control charts. Woodall (1986) identified this issue by noting that there is often a large number of false alarms due to the higher Type-I error probability in the economic design approach, in contrast to the statistical design, where the Type-I error is usually fixed. As such, Saniga (1989) considered economic-statistical design by placing some statistical constraints in terms of the in-control average run length (ARL_0) and the out-of-control average run length (ARL_1) on the cost model of the economic design to minimize the expected cost, where the effectiveness of control charts can be achieved by taking the statistical properties into account while the cost is minimized. Saniga's contribution inspired other researchers to make extensive developments in this area of study, such as Yeong, Khoo, Lee, and Rahim (2013), Liu, Yu, Ma, and Tu (2013), Lu, Huang, and Chiu (2013), Faraz and Saniga (2013), Mohammadian and Amiri (2013), Niaki, Gazaneh, and Toosheghanian (2014), Noorossana, Niaki, and Ershadi (2014), Yeong, Khoo, Lee, and Rahim (2014), Amiri, Moslemi, and Doroudyan (2015), Chiu (2015), and Yeong, Khoo, Yanjing, and Castagliola (2015).

Wu and Spedding (2000) introduced the synthetic control chart that combines the Shewhart $\overline{X}$ chart and the conforming run length (*CRL*) chart. In their paper, it was shown that the synthetic chart performs better than the $\overline{X}$ chart in detecting small to moderate mean shifts. A side sensitive synthetic chart was later proposed by Machado and Costa (2014). Gadre and Rattihalli (2004) made an extensive development on the group runs (GR) control chart, where this chart provides a better detection power than the synthetic chart for small and moderate shifts in the process mean. More recently, Gadre and Rattihalli (2007) developed the side sensitive group runs (SSGR) chart which surpasses the $\overline{X}$, synthetic and GR charts. It was highlighted that, the SSGR chart illustrates an important aspect of side sensitivity that the non-conforming subgroups should specify shifts on the same side of the target value of the process mean. A detailed explanation on the SSGR chart was presented in Gadre and Rattihalli (2007). Subsequently, Khoo, Tan, Chong, and Haridy (2015) introduced the side-sensitive group runs double sampling (SSGRDS) chart which improves the performance of the SSGR chart. Zhang, Xie, and Jin (2012) proposed an improved self-starting cumulative count of conforming chart for monitoring high-quality processes under group inspection. This chart presents some enhancements of the traditional group inspection procedures to improve the control charting technique for high-quality processes.

All existing economic and economic-statistical designs are based on the average run length (*ARL*), which requires the shift size to be known a priori. However, in actual situations, the shift size may not be known in advance. Thus, instead of computing the sets of optimal design parameters and the cost by considering the *ARL*, the minimization of costs for control charts should also be investigated based on the expected average run length (*EARL*). It is known that in practical conditions, the shift size is usually not deterministic and varies according to some unknown stochastic model. When data regarding the out-of-control process history are not obtainable, the quality practitioner must deal with the latter situation. Therefore, in this paper, the economic and economic-statistical designs of the SSGR control chart are formulated based on the *EARL*. The approach of using *EARL* as a performance measure has also been illustrated in Wu, Shamsuzzaman, and Pan (2004), Celano (2009) and Celano (2010). However, articles that adopt the *EARL* in the economic and economic-statistical designs cannot be found in the existing literature. To evaluate the overall cost effectiveness of the SSGR control chart, the optimal design procedures are demonstrated by considering both of the *ARL* and *EARL* as the components of the cost equation.

This paper is divided into several sections as follows, hereafter: Section 2 introduces the SSGR chart of Gadre and Rattihalli (2007). Section 3 presents the cost model of the economic and economic-statistical designs for the SSGR chart and the optimization algorithm in obtaining the optimal design parameters. This is followed by the results and sensitivity analyses in Section 4. In Section 5, the effects of misspecification of the shift size on the performance of the SSGR chart are studied. Section 6 shows the performance comparisons between the SSGR chart, the Shewhart $\overline{X}$ chart, the synthetic chart, the GR chart and the EWMA chart. Lastly, Section 7 concludes and summarizes the findings of this study.

2. The SSGR chart

For the process being monitored, it is assumed that the quality characteristic follows an underlying normal $N(\mu_0, \sigma^2)$ distribution, where μ_0 and σ^2 are the in-control mean and variance, respectively. The SSGR chart of Gadre and Rattihalli (2007) is characterized by combining the $\overline{X}/S$ sub-chart and an extended version of the *CRL*/*S* sub-chart. Gadre and Rattihalli (2007) made an extensive improvement to the GR chart by adding the side sensitivity feature, where this chart has a greater detecting power than the Shewhart $\overline{X}$, synthetic and GR charts. In this paper, the value of *CRL* for the *r*th non-conforming group is represented by Y_r for simplicity. The *CRL* is defined as the number of inspected groups between two consecutive non-conforming groups, including the non-conforming group at the end. The operation of the SSGR chart is explained in Gadre and Rattihalli (2007).

Gadre and Rattihalli (2007) obtained the out-of-control *ARL* (ARL_1) for the SSGR chart as

$$ARL_1 = \frac{[1-\alpha(1-\alpha)A^2]}{PA^2[1+\alpha(1-\alpha)(A-2)]}, \tag{1}$$

where $P = 1 - \Phi(k - \delta\sqrt{n}) + \Phi(-k - \delta\sqrt{n})$, $\alpha = \frac{1-\Phi(k-\delta\sqrt{n})}{P}$ and $A = P(Y_r \leqslant L) = 1-(1-P)^L$. Here, δ denotes the magnitude of the standardized mean shift (in multiples of standard deviation unit) and $\Phi(\cdot)$ is the standard normal cumulative distribution function (cdf). The in-control *ARL* (ARL_0) is obtained when $\delta = 0$.

For the condition of an unknown process shift size, the $EARL_1$ formula is given as

$$EARL_1 = \int_{\delta_{\min}}^{\delta_{\max}} f_\delta(\delta) ARL_1(\delta, L, k, n) d\delta, \tag{2}$$

where $EARL_0 = ARL_0$. $f_\delta(\delta)$ is the probability density function of the shift δ. In this paper, $f_\delta(\delta)$ is assumed to be a uniform density function over $(\delta_{\min}, \delta_{\max})$. There are two main reasons why the uniform distribution is adopted. Firstly, it is generally assumed explicitly (Domangue & Patch, 1991; Sparks, 2000) or implicitly (Reynolds &