Computers & Industrial Engineering 90 (2015) 371–380

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

A new stochastic approach for a reliable *p*-hub covering location problem

Marjan Sadeghi [a,*], Fariborz Jolai [a], Reza Tavakkoli-Moghaddam [a,b], Yaser Rahimi [a]

[a] *School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran*
[b] *Center of Excellence for Intelligence Based Experimental Mechanics, University of Tehran, Tehran, Iran*

ARTICLE INFO

Article history:
Received 24 May 2014
Received in revised form 20 September 2015
Accepted 6 October 2015
Available online 22 October 2015

Keywords:
Hub location problem
Stochastic programming
Degradable network
Capacity reliability
Meta-heuristics

ABSTRACT

A hub location problem (HLP) is a fertile research field, in the aspect of interdisciplinary studies, such as transportation, operation research, network design, telecommunication and economics. The location of hub facilities and allocation of non-hub nodes to hubs configure the backbone of HLPs. This study presents a new mathematical model for a reliable HLP by a new stochastic approach to minimize the total transportation cost and obtain maximum flows that the network can carry, when its link capacities are subject to stochastic degradations, as in a form of daily traffic, earthquake, flood, etc. We consider the road capacity reliability as a probability that ensures the maximum network capacity is greater than or equal to the total incoming flow to the network by considering the road capacity as random variable. As a result, this paper assumes that link capacities satisfy in a Truncated Erlang (*TErl*) distribution function. Due to complexity of the HLP, a meta-heuristic algorithm, namely differential evolution (DE) algorithm, is applied on the problem in order to achieve near-optimal solutions. Furthermore, the performance of the proposed algorithm (i.e., DE) is evaluated by the performance of the genetic algorithm (GA) applied on the given problem. Some computational experiments are presented to illustrate the effectiveness of the presented model and proposed algorithm. Finally the conclusion is provided.

1. Introduction

Hub location problems (HLPs) have risen when one deals with transferring data, passengers and goods between two subsets, namely origins and destinations. Hubs as critical and central elements benefit networks by concentrating, distributing and switching arrival flows instead of transferring flows between each origin–destination (O–D) pair directly. In fact, using hub facilities brings the advantages of economic scale to a network. Due to the above-mentioned reasons, nowadays HLPs are backbone of designing networks, such as designing transportation networks, postal delivery and telecommunication networks.

When the number of hub facilities is predefined and a specific radius is considered for each hub to cover the demand nodes, the HLP is called a *p*-hub covering location problem. There is a primary assumption at most of the related papers, that the hubs graph is a complete graph with high quality and more volume pathway. This assumption makes a situation to enable the discount factor $\alpha (0 \leqslant \alpha \leqslant 1)$ for all transportation costs of flows, which routed between each pair of hub facilities (Contreras, Fernandez, & Marin, 2009a; Eiselt & Marianov, 2009; Silva & Cunha, 2009). The classification of HLPs is formed by three criteria, namely type of space for locating hub-nodes, assignment rules and a number of hubs. In terms of the first criterion (i.e., type of space to locate hub-nodes), there are two classes, namely discrete location and continuous location. At the first class, hubs can be located only at specific and finite points while at the second class, hubs can be located arbitrarily in a region. In terms of the second criterion (i.e., assignment rules), there are two classes entitled by single allocation and multiple allocation. At the first class, non-hub nodes as demand points are assigned to only one hub. By contrast, at the second class, non-hub nodes as demand points can be assigned to more than one hub. In terms of the third criterion, the number of hubs in a trip is limited to one or two hubs, namely 1-stop and 2-stop, respectively. Classical objective functions try to minimize the total cost, minimize the maximum of traveling time, and so forth. However, there is not enough variety in concept of the objective function. Some universal surveys in this scope are provided by Campbell, Ernst, and Krishnamoorthy (2002), Alumur and Kara (2008), Campbell and O'Kelly (2012), Zanjirani Farahani, Hekmatfar, Boloori Arabani, and Nikbakhsh (2013).

Efforts for an effective model of the transportation process rise for having the better transportation systems in the aspect of

* Corresponding author. Tel.: +98 21 88021067; fax: +98 21 88013102.
E-mail address: marjan_sadeghi@ut.ac.ir (M. Sadeghi).

http://dx.doi.org/10.1016/j.cie.2015.10.003
0360-8352/

economic, environmental and efficiency concerns (Lo, Luo, & Siu, 2006). According to aging transportation infrastructure; threat of security concerns; unavoidable occurrence of natural disasters and traffic incidents, studding the better mathematical modeling for transportation systems in degradable networks, gets high importance. Focusing on modeling, the transportation network by considering degradations on the network is not a negligible issue in the aspect of many science branches, such as industrial engineering, urban management, and operation research.

In the real world, after designing a transportation network, some parameters (e.g., costs, demands, transportation time and road capacity) of the given problem may change due to uncertain environment. For example, in degradable networks, degradation factors cause some degradation on road performances (Dianhai, Hongsheng, & Cheng, 2010). This disruption in the network reduces the road capacity and increases the traveling time uncertainty (Lo & Tung, 2003). The variability in road capacity and traveling time puts the network in an uncertain situation. There are many papers about uncertain HLP, which are mentioned in Section 2.1.

Nowadays, the study on the road capacity reliability is increasing significantly, and it is one of the most important research topics. Reliable transportation systems are important not only in the aspect of travelers, but also in aspect of traffic managers. Some evidence (e.g., recent earthquakes in Taiwan, Japan and United States) are obvious reminders of the significant impacts of road degradations on the transportation network. On a day-to-day, both minor scale and significant scale disruptions occur in terms of traffic incidents and natural disasters on a transportation network. As a consequence, these disruptions reduce the in used link capacities and cause to degrade the road network performance (Lo & Tung, 2003). As a result, it is notable to estimate and quantify these impacts, which can help to improve the incident management strategies and the network design.

Capacity reliability is defined as a probability that a network can accommodate certain demands at a specific given service level (Chen, Yang, & Lo, 1999, 2002). Another approach exists for defining the capacity reliability as the maximum flow, which the network can carry by considering the link capacity constraints (Lo & Tung, 2000, chap. 11). The most important part of this matter relies on chance-constraint programming (Charnes & Cooper, 1963). In the other words, the capacity reliability is a probability that ensures the maximum network capacity is greater than or equal to the total incoming flow to the network by considering the road capacity as random variations. A chance-constraint approach is used extensively at environmental planning, water system design and utility distribution (e.g., see Guldmann, 1983; Mays & Tung, 1992; Jacobs, Medina, & Ho, 1997). In fact, this definition of the chance-constraint approach converts the probabilistic reliability requirements to the deterministic format, and at last it forms specific constraint sets of the mathematical model directly. This procedure produces a mathematical program, which is ready to solve and achieve an optimal solution. This chance-constraint approach is a basic concept of this paper, which is discussed comprehensively in Section 3.

2. Literature review

Some motivations for absorbing researches to focus on HLPs emerge in the 1970s and 1980s. In this regard, we can mention to transportation regulation, increase of express delivery services, development of large telecommunications network and enhancement in modeling. The pioneering model within HLPs field was presented by O'Kelly (1986) as quadratic mathematical formulation. After that, Campbell (1994) extended a linear model based on the quadratic model of O'Kelly (1986). This model is well-known as p-hub median problem. Ernst and Krishnamoorthy (1998) presented a more practical model than earlier models by reducing the number of variables and constraints in this scope. Skorin-Kapov, Skorin-Kapov, and O'Kelly (1996) reached to exact solutions of a p-hub median problem by boosting the linear relaxation introduced by Campbell (1994). Hamacher, Labbe, Nickel, and Sonneborn (2004) used polyhedral properties from a facility location scope to create a new mathematical model. There are complete surveys, as we mentioned in Section 1 for more detailed information in this regard. Following, we try to mention papers with the uncertain HLP concept.

2.1. Uncertainty in hub networks

Yang, Liu, and Yang (2013a) presented a new fuzzy p-hub center problem and assumed travel time as uncertain parameter characterized by a normal fuzzy vector. They considered a certain efficient traveling time point for each path on the network. Also, the objective function of the model tried to maximize credibility of fuzzy travel time in a way that not exceed from the predetermined travel time point. To reformulate the main problem to mixed-integer model and discretize the fuzzy travel time, they used an approximation approach. Then, they extended a parametric decomposition to divide the approximated model into two mixed-integer programming sub-problems. They extended a partial swarm optimization (PSO) algorithm to tackle the problem for large-sized instances.

Bashiri, Mirzaei, and Randall (2013) presented a capacitated p-hub center problem and considered both quantitative parameters (e.g., cost and time) and qualitative parameters (e.g., quality of service, zone traffic, environmental issues and capability for development in the future) in their mathematical model. Also, some parameters are assumed as uncertain parameters. Then, they used a hybrid fuzzy approach (i.e., VIKOR) to the presented HLP. Also, they solved the mathematical model by applying the genetic algorithm and by using the AP data set. Yang, Liu, and Yang (2013b) presented a new risk-aversion p-hub center problem and considered traveling time as uncertain parameter with a fuzzy approach. They also reformulated the presented model to a parametric mixed-integer programming problem. To achieve the near-optimal solution, they used a hybrid algorithm based on GA and local search.

An, Zeng, Zhang, and Zhao (2014) proposed a reliable p-median facility location network by considering disruption in the network. Also, they solved the presented model by column-and-constraint generation method. Taghipourian, Mahdavi, Mahdavi-Amiri, and Makui (2012) studied a dynamic virtual HLP and assumed some disruption in the hub facilities, in which it affects the entire network. Accordingly, they presented a fuzzy integer linear programming. Also they evaluated their model by using CAB data sets. Kim (2012) introduced new hub and spoke survivable network designs by considering disruption in the network. These models are termed as *p-hub protection models* (PHPRO). The PHPRO models try to maximize the total potential interacting traffic over a set of origin–destination nodes by considering different routing assumptions. Snyder (2006) provided a comprehensive review about facility location under uncertainty.

2.2. Stochastic approach

Marianov and Serra (2003) presented an M/D/c HLP at an airport center. A mixed-integer linear programming (MILP) model was adapted to define the problem. Also, they used a chance-constraint method to limit the queue length, which was formed by airplanes. A heuristic approach was used to obtain feasible