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A location–inventory supply chain problem: Reformulation and piecewise linearization

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ABSTRACT

In this paper, we study a two-echelon inventory management problem with multiple warehouses and retailers. The problem is a natural extension to the well-known one-warehouse multi-retailer inventory problem. The problem is formulated as a mixed integer non-linear program such that its continuous relaxation is non-convex. We propose an equivalent formulation with fewer non-linear terms in the objective function so that the continuous relaxation of the new model is a convex optimization problem. We use piecewise linearization to transform the resulting MINLP to a mixed integer program and we solve it using CPLEX. Through numerical experiments, we compare the solutions obtained by solving the new formulation using CPLEX with two previously published Lagrangian relaxation based heuristics to solve the original mixed integer non-linear program. We demonstrate that the new approach is capable of providing almost the same solutions without the need of using specialized algorithms. This important contribution further implies that additional variants of the problem, such as multiple products, capacitated warehouses and routing, can be added to result in a problem that will again be solvable by commercial optimization software, while the respective Lagrangian heuristics will fail to solve such variants or extended problems.

1. Introduction

In a highly competitive market, it becomes a necessity for companies and organizations to optimize their supply chain in order to ensure efficient operations, which will lead to lower costs and higher customer satisfaction. In order to reduce costs and increase service levels, effective supply chain strategies should be implemented, that take into consideration the interactions at the different stages of the supply chain. Therefore, supply chain management involves decisions to be made on various levels: strategic decisions, tactical decisions, and operational decisions (Dolgui & Proth, 2010; Simchi-Levi, Kaminsky, & Simchi-Levi, 2003).

Strategic-level decisions refer to planning that impacts the company over a long period of time, such as the location of facilities (warehouses), while tactical-level decisions have a short-term impact, relative to the strategic decisions, as they determine factors more frequently, such as transportation and inventory policies. Finally, the operational-level decisions are performed on a weekly or daily basis in order to plan the scheduling and routing. In terms of strategic-level decisions, one of the most important problems is the facility location problem, while amongst tactical-level decisions one of the most important is the inventory policy problem. These problems are interdependent, in the sense that modifying the number or location of warehouses has an impact on inventory costs, and similarly a change in inventory policy impacts assignment decisions and thus also affects location-related costs. However, these two problems are usually dealt with independently, which leads to sub-optimal solutions.

Considering the two decision levels in an integrated manner results in the joint location–inventory problem, which can lead to improved management of the supply chain, as more information is utilized and the important interactions between the problems are captured (Diabat, Richard, & Codrington, 2013). The basic model addresses the delivery of a single product from a manufacturer to warehouses, also named distribution centers (DCs), and from there to multiple retailers. The DCs can be opened in multiple locations, which is something that is decided through the model. DCs and retailers hold working inventory, based on product that has not yet been requested by retailers or end-customers, respectively. The purpose of the model is to determine the optimal number of open DCs, the optimal allocation of retailers to DCs, and the optimal inventory strategies for both DCs and retailers. The objective is to minimize the fixed ordering, holding, transportation and facility-related costs. This problem was addressed by Teo and Shu

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(2004), Shu (2010), Diabat et al. (2013), Diabat, Battaïa, and Nazzal (2015).

Integration, however, of the location and inventory problem comes at the cost of higher model complexity, leading to the need for use of heuristics. Such is the case in the works of Chan, Chung, and Wadhwa (2005), Jayaraman and Ross (2003), Diabat, Aouam, and Ozsen (2009), Shiguemoto and Armentano (2010), and Diabat (2014) who developed metaheuristics such as genetic algorithms, simulated annealing and tabu search to solve similar problems. Furthermore, Lagrangian relaxation has been implemented by Chen and Chu (2003), Diabat and Richard (2015), and Eskigun et al. (2005). Benders decomposition method is used by Santibanez-Gonzalez and Diabat (2013) for solving a reverse logistics problem.

In the current paper, we focus on the multi-echelon joint location–inventory problem, which decides on the locations of warehouses, through which a single product will be distributed from the manufacturer to the retailers. The model assigns retailers to warehouses and determines the times between orders at the warehouses and retailers in order to minimize the operating cost of the supply chain. We evaluate the performance of the developed technique by comparing with the results obtained by previous works by Diabat et al. (2013) and Diabat et al. (2015), who develop Lagrangian heuristics for the problem. The objective of this paper is to address the integrated problem in a way that allows for solving it with the use of commercial software, rather than through the development of complicated heuristic techniques, which is prevalent in the literature. The additional benefit of this is that possible extensions to the model can be made so that the model remains solvable by commercial software. On the other hand, the Lagrangian heuristics with which we compare the particular model, will more likely fail to solve the extended problems.

2. Literature review

Most literature has traditionally considered facility location decisions and inventory management decisions independently. The motivation to solve integrated strategic and tactical level problems in order to improve supply chain management is reflected in more recent streams of research.

A study of a multi-echelon join inventory–location model that produces the simultaneous decision of warehouse location and inventory policies at both the warehouses and retailers was performed by Diabat et al. (2013). They formulate the model as a non-linear mixed-integer program and solve it using a Lagrangian relaxation-based heuristic. An important observation the authors make is that for certain regions of the parameter space there is an undoubtable benefit for the integration. Later work by Diabat et al. (2015) develops an improved Lagrangian relaxation-based heuristic for the same problem. The approach is based on reducing the solution space by removing feasible solutions that can't be optimal before implementing the Lagrangian relaxation heuristic.

Another interesting approach is that of Teo and Shu (2004), who structure the location–inventory design problem as a set-partitioning integer programming model and solve it using column generation, in order to determine the number of open warehouses, their location, the allocation of retailers to these warehouses and finally the optimal inventory policies for both entities. Their results demonstrate that moderate size problems can be solved within reasonable time. In later work, Shu (2010) develops a simple greedy algorithm for the set-covering problem, which selects the best set at each iteration, until all elements are covered by the selected sets. Because of the large number of sets, the author reduces the sub-problem to a sub-modular function minimization problem. Results show that the greedy algorithm can solve large scale problems with satisfactory performance, since solution errors are within 3–4%.

Daskin, Coullard, and Shen (2002) and Shen, Coullard, and Daskin (2003) developed a location–inventory model with risk pooling (LRMP), which is formulated as a non-linear integer programming problem that incorporates inventory costs at the DCs. Risk pooling was also addressed by Vidyarthi, Çelebi, Elhedhli, and Jewkes (2007), who consider a multi-product two-echelon production–inventory–distribution system. Their model incorporates risk-pooling effects by consolidating the safety-stock inventory of the retailers at DCs. It is formulated as a non-linear mixed-integer program and it is linearized using piecewise-linear functions. The authors decompose the problem by echelon using Lagrangian relaxation, which provides a lower bound and a heuristic is proposed to produce overall feasible solutions. Their results prove that the Lagrangian relaxation provides a tight lower bound, while the heuristic solution is within 5% of the optimal.

As a final note in this section, one might argue that since there are several developed heuristic techniques for addressing this problem, which perform extremely well, why an approach such as the current one, that allows for solving through commercial software, may be useful. The reason is that in potential extensions of the model, techniques such as Lagrangian relaxation will more likely fail to provide a solution, whereas with the current approach it is easy and straightforward to incorporate the extension and again solve the problem with the help of commercial software. Once again, we highlight this important contribution of the current work.

3. Problem description and reformulation

3.1. Problem description

Given a set of retailers I, and a set of warehouses that can be located at certain predetermined sites J, the aim of the multi-echelon joint inventory–location problem is to distribute a single commodity from a single manufacturer to the warehouses J and from there to the retailers I. The retailers deal with deterministic demands and they hold working inventory, which is the product that has been ordered from a warehouse but has not yet been requested by end-customers. As far as the warehouses are concerned, they order a single commodity from the manufacturer at regular time intervals and they distribute the product to retailers. For the warehouses, the working inventory represents product that has been ordered from the manufacturer, but has not yet been requested by retailers. In this model, lateral supply among warehouses is prohibited, as warehouses are only supplied by the manufacturer, and shortages are not permitted. Note also, that the existence of the production unit does not imply a three-echelon model. Rather, it demonstrates that the shipping cost between production and retailers is accounted for, in order to aid in the decision of which warehouses will be opened, as will become evident in subsequent sections through the model formulation. A graphic illustration of the described network can be seen in Fig. 1.

At this point it is important to discuss the adopted assumption of the power-of-two inventory policy (Roundy, 1985; Simchi-Levi, Chen, & Bramel, 2007). We begin by defining the cycle time for each echelon, which is the time starting when the work begins upon a specific request and ends when the item is ready for delivery. Thus, for example, the cycle time for distribution commences when they request an item from the manufacturer and ends when this item is ready to be delivered to the retailer. Applying common production cycle policy implies that the production cycle times of all end items are identical. However, adopting a power-of-two policy assumes that the ratio of the cycle times is a power-of-two number. This