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Optimization of variable demand fuzzy economic order quantity inventory models without and with backordering ☆

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ABSTRACT

A variable demand inventory model was developed for minimizing inventory cost, treating the holding and ordering costs and demand as independent fuzzy variables. Thereafter, backordering cost was also considered as an independent fuzzy variable. Fuzzy expected value model and fuzzy dependent chance programming model were constructed to find the optimal economic order quantity, which would minimize the fuzzy expected value of the total cost, so that the credibility of the total cost not exceeding a certain budget level was maximized. Optimization was carried out using genetic algorithms and particle swarm optimization algorithm, and their performances were compared. The developed model was found to be efficient not only in one artificial case study but also in two data sets collected from the industries. Therefore, this model could solve real-world problems, too.

1. Introduction

Economic order quantity (EOQ) is the optimal order quantity that minimizes total inventory cost (which includes holding, ordering and backordering (if any) costs). Control of inventory is necessary to minimize expenses. The model was developed by Harris in 1913 (Harris, 1913), and extensively applied by Wilson (1934). The underlying assumptions in basic EOQ model is that demand for a product is known and constant, and that each new order is delivered instantaneously, whenever inventory reaches to zero. Holding cost is taken as a certain percent of raw material cost. Ordering cost is kept fixed and considered to be independent of the number of units ordered.

In industry, the costs of holding, ordering and backordering are always likely to vary from one cycle to another. Demand may also vary from time to time. Absence of historical data makes it difficult to estimate the probability distribution of these variables. Thus, fuzzy set theory, rather than the traditional probability theory, is better suited for analysis of inventory. The investigators had carried out a study for fixed demand, treating holding cost, ordering cost and backordering cost (if any) to be fuzzy in nature (Samal, Kumar, & Pratihar, 2014). In the present study, demand too was taken to be fuzzy in nature, and real data from industry were utilized to validate the model. Zadeh (1965) introduced fuzzy set theory in 1965 and the same was utilized to develop realistic models of inventory control. EOQ problem was investigated by introducing the concept of fuzzy set theory to an inventory system. The model was optimized using both genetic algorithms (GA) and particle swarm optimization (PSO) algorithm.

2. Literature review

Control of inventory is one of the important areas of operation management. A proper control of inventory can significantly bring down operating expenses and increase profits. The economic order quantity (EOQ) formula was introduced by Harris (1913) in 1913. Fuzzy set theory, introduced by Zadeh (1965) in 1965, had been utilized for realistic modeling of inventory control system. A discussion on attempts by various investigators to study and optimize inventory models, is presented below.

Kacprzyk and Staniewski (1982) applied the fuzzy set theory to inventory control problem and considered a long term inventory policy making through fuzzy decision models in 1982. Inventory control by optimal policies for controlling cost rates in a fluctuating demand environment was investigated by Song and Zipkin (1993) in 1993. Uncertainty in inventory control had been modeled utilizing fuzzy knowledge-based system by Petrovic and Sweeney (1994) in 1994. Fuzzy numbers were used to represent uncertain data and relations between them modeled by fuzzy if–then rules. Variation in demand due to causes like economic recession etc was modeled by Brill and Chaouch (1995) in 1995.

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Nomenclature

A	an event which is basically each element of $\mathcal{P}(\Theta)$
A^c	compliment of A, i.e., not occurrence of A
B	backorder quantity, taken as some fixed proportion of EOQ
c_1, c_2	first and second communication coefficients in a PSO algorithm
$Cr\{A\}$	a number assigned to each A, which denotes the credibility that A will occur
D	total demand in the planning period
G	number of generations of a GA
h	inventory holding cost in Rs. per year
k	cost of placing one order in Rs.
n	number of runs of a PSO algorithm
N	population size of a GA
p_c	probability of crossover of any two selected chromosomes in a GA
p_i	best position vector of a given particle in PSO algorithm
p_g	position vector of the fittest particle in PSO algorithm
p_m	probability of mutation of any given chromosome in a GA
$Q^{\star}$	economic order quantity
R	preallocated budget limit for procurement and holding of inventory
S	swarm size of PSO algorithm
t	cycle time of one inventory procurement
T	length of planning period
TC	total variable cost towards procurement and holding of inventory
V_i	velocity vector of a particle in PSO algorithm
w	inertia weight in PSO algorithm
X_i	position vector of a particle in PSO algorithm
μ	membership function value of a fuzzy parameter
Θ	non empty set
ξ	fuzzy variable
$\mathcal{B}$	any set of real numbers, such that $\mathcal{B} \in \mathcal{R}$
$\mathcal{P}(\Theta)$	power set of Θ (i.e., all the subsets of Θ)
$\mathcal{R}$	set of real numbers
EOQ	economic order quantity
FDCP	fuzzy dependent chance programming
GA	genetic algorithms
PSO	particle swarm optimization

In 2003, Wu and Yao (2003) studied fuzzy inventory models with backorder for fuzzy order and shortage quantities. Triangular fuzzy membership function distributions were used in their study, and it was seen that fuzzification of both order and shortage quantities could give better results than fuzzifying any one variable. Kao and Hsu (2002), Dutta, Chakraborty, and Roy (2005) studied single period inventory model with fuzzy demand and fuzzy random variable demand, respectively, and developed models for optimum order quantity in terms of cost. In 2003, Yao and Chiang (2003) investigated inventory model without backorder with fuzzy total cost and storing cost. Trapezoidal and triangular fuzzy numbers were used in their investigation.

Fluid models were used for optimization of inventory models dealing with demand uncertainty by Adida and Perakis (2006). Backorder was not included in their model. Gao and Feng (2006) investigated multiple commodities as dynamic programming model, where genetic algorithms followed by neural networks were trained to approximate the optimal cost function on a randomly generated sample set in 2006. A literature survey on models for production planning under uncertainty till 2006, was carried out by Mula, Poler, Garcia-Sabater, and Lario (2006).

Syed and Aziz (2007) modeled inventory model without shortage under fuzzy environment in 2007. Ordering and holding costs were considered as fuzzy triangular numbers, and optimum order quantity found out using signed distance method. Gallego, Katricioglu, and Ramchandran (2007) analyzed variation of base stock levels for inventory under normal, log-normal and gamma and negative binomial distributions type demand. In 2007, Wang, Tang, and Zhao (2007) developed the model of fuzzy economic order quantity without backordering. Holding cost and set-up cost were considered as fuzzy in nature and the model was developed for keeping the credibility of total cost in the planning period below certain budget level. Vijayan and Kumaran (2008) investigated continuous review and periodic review inventory models under fuzzy environment, where the membership function distribution took a trapezoidal form.

In 2009, Chou (2009) proposed a fuzzy economic order quantity (FEOQ) inventory model. Trapezoidal fuzzy numbers were used to represent costs and quantities. Mahata and Goswami (2009) investigated economic order quantity (EOQ) taking the demand rate, lead-time and inventory costs as fuzzy numbers in 2009. The products, whose demand can be manipulated by artificially restricting supply were studied by Sapra, Truong, and Zhang (2010). Multi-item mixture inventory model, in which both demand and lead time were random, along with a budget constraint was investigated by Bera, Rong, Mahapatra, and Maiti (2009). Fuzzy chance-constrained programming technique was utilized in their investigation.

Fuzzy inventory model of deteriorating items was carried out by Jadhav and Bodkhe (2010), however, they did not include shortages in their model. Singh, Kumar, and Gupta (2011) and Malik and Singh (2011) utilized soft computing techniques for modeling of inventory under price dependent demand and variable demand, respectively. Halim, Giri, and Chaudhuri (2011) developed production planning models for fuzzy demand rate. Trapezoidal fuzzy numbers were considered in the analysis. Wang (2011) investigated a mixture inventory control system, in which various parameters like lead time demands in different cycles, defective rates of arrived order, various ordering costs etc. were considered to be independent and identically distributed random variables. A fuzzy simulation algorithm and an iterative algorithm were designed to minimize the expected total annual cost. In the same year, Mahata and Mahata (2011) applied fuzzy EOQ model to supply chains and Rong (2011) developed EOQ model by treating the holding cost, shortage cost and ordering cost per unit as uncertain variables.

Variation in demand due to fuzziness and randomness was modeled by Nagare and Dutta (2012) in 2012. They developed an expert system, which could determine the optimal order quantity, so that the profit was maximized. Kumar, De, and Goswami (2012) developed fuzzy EOQ models with ramp type demand after taking deterioration of stocked items into account. Fuzzy EOQ model with imperfection, shortages and inspection errors was developed by Liu and Zheng (2012), whereas, Saha and Chakrabarti built fuzzy EOQ model for deteriorating items (Saha & Chakrabarti, 2012).

Bjork (2012) investigated uncertainty in industry utilizing asymmetric triangular fuzzy numbers. Hsu and Hsu (2013)