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Heuristic for the two-dimensional arbitrary stock-size cutting stock problem

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ABSTRACT

A heuristic is presented for the two-dimensional arbitrary stock-size cutting stock problem, where a set of rectangular items with specified demand are cut from plates of arbitrary sizes that confirm to the supplier's provisions, such that the plate cost is minimized. The supplier's provisions include: the lengths and widths of the plates must be in the specified ranges; the total area of the plates with the same size must reach the area threshold. The proposed algorithm uses a pattern-generation procedure with all-capacity property to obtain the patterns, and combines it with a sequential heuristic procedure to obtain the cutting plan, from which the purchasing decision can be made. Practical and random instances are used to compare the algorithm with a published approach. The results indicate that the trim loss can be reduced by more than half if the algorithm is used in the purchasing decision of the plates.

1. Introduction

In the two-dimensional arbitrary stock-size cutting stock problem (2D_ASSCSP), m types of rectangular items with length l_i, width w_i and demand d_i, $i = 1, \ldots, m$, are cut from plates of arbitrary sizes that confirm to the supplier's provisions, such that the plate cost is minimized. The solution is a cutting plan consisting of K different cutting patterns, where the kth pattern P_k has plate size $L_k \otimes W_k$ (length $\otimes$ width) and frequency x_k, $k = 1, \ldots, K$. The items in a pattern are placed orthogonally and without overlap. They are not oriented, that is, either the length or width of an item can be in horizontal direction. The 2D_ASSCSP can be formulated as:

$$\max \sum_{k=1}^{K} S_k x_k \tag{1-0}$$

$$\sum_{k=1}^{K} a_{ik} x_k = d_i, \quad i = 1, \ldots, m; \tag{1-1}$$

$$L_k \in [L_{\min}, L_{\max}], W_k \in [W_{\min}, W_{\max}], \quad k = 1, \ldots, K; \tag{1-2}$$

$$\sum_{j=1}^{K} \tau_{jk} S_j x_j \geqslant S_{\min}, \quad k = 1, \ldots, K; \tag{1-3}$$

$$\tau_{jk} = 1 \text{ if } (L_j = L_k) \cap (W_j = W_k),\ \tau_{jk} = 0 \text{ otherwise},\ j, k = 1, \ldots, K; \tag{1-4}$$

$$x_k \in \mathbf{N}, \quad k = 1, \ldots, K; \tag{1-5}$$

where:

S_k	area of plate $L_k \otimes W_k$, $S_k = L_k W_k$, $k = 1, \ldots K$.
a_{ik}	number of type-i items in the kth pattern, $i = 1, \ldots, m$, $k = 1, \ldots, K$.
$L_{\min}$	minimum plate length allowed.
$L_{\max}$	maximum plate length allowed.
$W_{\min}$	minimum plate width allowed.
$W_{\max}$	maximum plate width allowed.
$S_{\min}$	area threshold. Total area of the plates of the same size must reach $S_{\min}$.
$\mathbf{N}$	set of non-negative integers.

The objective is to minimize plate cost (area). Constraint (1-1) reflects that the items demands must be met exactly. Constraint (1-2) denotes that the plate sizes must be in the specified ranges. Constraint (1-3) indicates that the total area of the plates with the same size must reach the area threshold. Constraint (1-5) requires that the frequencies of the patterns be non-negative integers.

The 2D_ASSCSP appears in a project with the following features:

(1) The total weight of the steel plates used is large (say, more than 100 tons). Then the supplier is willing to produce the plates according to the customer's specifications because of the considerable gain from the large purchase.
(2) It is not adequate to share the purchase among projects.

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(3) The sizes of some items are too large to be produced from plates of ordinary sizes that can be purchased from stores in the market.

Such projects include the steel-bridge construction (Cui & Lu, 2009). It may also include the stadium construction and the building of large ships. Fabrication of structural metals is an important stage in such projects. It consists of operations necessary to prepare members or groups of members for use in structures. The required materials should be purchased before shop operations. The blanks (items) of most members made of steel plate are rectangular. Some items have too large sizes. This makes it impossible to produce them from ordinary-size plates that can be obtained from stores in the market. Subsequently the contractor just purchases all necessary plates from the producer (supplier), because price discount often exists for large order quantities. The plate sizes in the purchasing order should confirm to the supplier's provisions that include the size ranges $[L_{\min}, L_{\max}]$, $[W_{\min}, W_{\max}]$ and the area threshold $S_{\min}$, where the size-ranges are usually determined from considering easy transportation and production specification, and the area threshold from considering the setup cost.

The purchasing order is known once the cutting plan is generated. This paper presents a heuristic (GBA, where GB denotes that General Block patterns are used and A represents Algorithm) for the 2D_ASSCSP. The GBA considers a specified number of cutting plans, from which the best one is taken as the solution. The patterns in each cutting plan are generated sequentially using a pattern-generation procedure (GETPAT). Arbitrary plate sizes can be considered because GETPAT has the all-capacity property. Both practical and random instances are used in the computational test to compare the GBA with a published algorithm. The results show that the GBA is able to reduce the trim loss by more than half.

The following assumptions are further made considering the features of the practical backgrounds of the 2D_ASSCSP and the non-orientated items:

$$W_{\min} < W_{\max} \leqslant L_{\min} < L_{\max};\ l_i \geqslant w_i,\ i = 1, \ldots, m;\ L_{\min}W_{\min} \leqslant S_{\min} < L_{\max}W_{\max}.$$

The remainder of this paper is arranged as follows: The literature is reviewed in the next section. Algorithm GBA for generating the cutting plan is described in Section 3, followed by the description of the GETPAT in Section 4. Implementation issues are discussed in Section 5. The computational results are described in Section 6, and the conclusions are given in the final section.

Global (used across sections/sub-sections) symbols and abbreviations are listed in Appendix for quick reference. They are also defined where they appear the first time in this paper.

2. Literature review

In the two-dimensional cutting stock problems (2DCSP), a set of rectangular items with specified demand are cut from stock plates to minimize material cost. In Wäscher, Haußner, and Schumann (2007), the cutting stock problems are classified into three subtypes according to the features of the plates. They are respectively the single stock size cutting stock problem (SSSCSP), multiple stock size cutting stock problem (MSSCSP) and residual cutting stock problem (RCSP). These problems are denoted respectively as the 2D_SSSCSP, 2D_MSSCSP and 2D_RCSP in this paper to emphasize that they are two-dimensional.

All stock plates have the same size in the 2D_SSSCSP. Recent algorithms for this problem include (Macedo, Alves, & Valério de Carvalho, 2010; Silva, Alvelos, & Valério de Carvalho, 2010; Cui, 2004; Cui, 2012; Suliman, 2006; Cintra, Miyazawa, Wakabayashic, & Xavier, 2008). Macedo et al. (2010) presented an exact arc-flow model for the case where two-staged patterns are used. Silva et al. (2010) proposed an exact algorithm for the case where only two- and three-staged patterns are allowed and the items are oriented. Although the restrictions on the pattern types and item orientation are useful to reduce the solution space, the exact algorithms cannot obtain the optimal solutions for middle- and large-scale instances because of the long computation time. The linear programming (LP) approach is used in Cui (2004), Cui (2012). The decision variables are the frequencies of the cutting patterns. They may be fractional in the optimal LP solution and must be rounded to integers. Although the LP approach is adequate for solving large-scale instances, the solutions may have large gap to optimality when the average demand of an item type is small, because of rounding errors. Suliman (2006) described a SHP-based algorithm, where SHP stands for sequential heuristic procedure. The SHP generates a pattern to fulfill some portion of the remaining demands, and repeats until all demands are met. The performance of the algorithm is not known because no computational results are given, except the solution to a small illustrative example. Cintra et al. (2008) presented a hybrid approach that solves the 2D_SSSCSP in two phases. The residual problems are solved repeatedly in the first phase, using the LP approach. The first residual problem is the original problem. The fractional frequencies in the LP solution of the current residual problem are simply rounded down to integers, producing a new residual problem. The first phase is terminated when the rounding procedure produces zero frequencies for all patterns. Then a heuristic strategy is used in the second phase to solve the last residual problem. This approach may leads to solutions close to optimal.

The stock plates in the 2D_MSSCSP have more than one stock size. The assortment of the stock plates is weekly heterogeneous, that is, the supply quantities of the plate types are relatively larger, compared to those in the 2D_RCSP. The number of published algorithms for the 2D_MSSCSP is relatively small. Alvarez-Valdes, Parajon, and Tamarit (2002) presented several LB-based heuristics for the 2D_MSSCSP. The LP approach solves the 2DCSP iteratively. In each round of iteration, a 2DSLOPP that denotes the two-dimensional Single Large Object Placement Problem (Wäscher et al., 2007) is solved to obtain a pattern on a stock plate. The heuristics in Alvarez-Valdes et al. (2002) differ in the methods used to solve the 2DSLOPP. A SHP-based heuristic is available in Al-Bouri (2006).

The 2D_RCSP denots a cutting stock problem with a strongly heterogeneous assortment of plates (Wäscher et al., 2007). In practice the 2D_RCSP comes about whenever the plates are to be used which represent leftovers generated in previous cutting processes. Here *strongly heterogeneous assortment* means that the supply quantities of most plate types are small. Algorithms dealing with the 2D_RCSP are rare. A nested decomposition approach for the 2D_RCSP is available in Vanderbeck (2001), where homogenous three-staged patterns (Cui, 2008) are used.

It is seen that the 2D_ASSCSP discussed in this paper does not belong to any of the three 2DCSP subtypes, because the sizes of the plates are not known and should be determined through optimization. A specialized heuristic approach for solving the 2D_ASSCSP in steel-bridge constriction is available in Cui and Lu (2009). Here *specialized* is used because it is assumed that the items have special features. The cutting plan is generated in two stages. In the first stage of the solution process, the approach generates five types of simple patterns quickly, where each type accommodates only items with the same feature. Layer patterns are used in the second stage. Although it is time-consuming to generate a layer