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A new exponentially weighted moving average control chart for monitoring the coefficient of variation ☆

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ABSTRACT

Monitoring coefficient of variation is one of the successful approaches to Statistical Process Control (SPC) when the process mean and standard deviation are not constants. This paper presents a modified Exponentially Weighted Moving Average (EWMA) chart in order to further enhance the sensitivity of the EWMA control chart proposed by Castagliola et al. (2011). Tables are provided for the statistical properties of the new chart. Some numerical results and comparisons are given and show that the new chart has an average run length performance that is superior to some other competing procedures. A real data example from manufacturing shows that it performs quite well in applications.

1. Introduction

Ever since Shewhart introduced control charts, it has become a common practice for practitioners to use various control charts to monitor different processes. When we deal with variable data, the charting technique usually employs a chart to monitor the process mean and another chart to monitor the process variance. The Shewhart $\overline{X}$ and S (or R) charts are industry standards for quality control applications where the mean μ and the standard deviation σ of a process must be statistically controlled at the nominal values μ_0 and σ_0. The baseline assumption is that the nominal values are fixed constants, and there are indeed many applications for which this assumption is reasonable. To this end, it is reasonable to monitor the process mean and variance simultaneously by a single chart, see Zhang, Zou, and Wang (2010, 2011), Costa and Machado (2013), Du, Huang, and Lv (2013) and Menzefricke (2013a, 2013b). However, control charting techniques were recently extended to various service sectors such as health, education, finance (see Sharpe, 1994) and various societal applications. In addition, it is also adopted in chemical and biological assay quality control to validate results, where the mean and the standard deviation may not be constants all the time and the process may nevertheless be declared in-control if their ratio remains stable around a constant value, see Reed, Lynn, and Meade (2002). As stated by Castagliola, Achouri, Taleb, Celano, and Psarakis (2013a, 2013b), there are many opportunities for SPC monitoring of the coefficient of variation (CV) also in the fields of materials engineering and manufacturing. Tool cutting life and several properties of sintered materials are typical examples from this setting, and hence we will show our proposed scheme performs quite well in applications through a real data example from sintered materials manufacturing. In this case, the routine use of the Shewhart charts is dubious, even though statistical control is still sought. For example, direct proportionality $\sigma = \gamma\mu$ is a common relationship between the mean and standard deviation in some processes. In this less restrictive setting, μ and σ may vary in the parameter space subject only to $\gamma = \frac{\sigma}{\mu}$, so that only the CV parameter, γ, is constant. In this case, it is natural to explore the use of the CV. Several published works have investigated the distribution of sample CV and its related inferential properties, see Hendricks and Robey (1996), Iglewicz, Myers, and Howe (1968), Mckay (1932), Mahmoudvand and Hassani (2009), Reh and Scheffler (1996), Tian (2005), Vangel (1996) and Verrill and Johnson (2007).

Recently, Kang, Lee, Seong, and Hawkins (2007) developed a Shewhart-Type control chart for monitoring the cyclosporine level in organ-transplantation procedures using rational subgroups. As stated by Kang et al. (2007), the advantage of adopting $\hat{\gamma}$ as the monitored statistic by a control chart is evident for those chemical or physical processes for which the variation of a quality characteristic X has to be controlled and the population standard deviation σ is proportional to the mean μ. This Shewhart-Type chart is sensi-

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tive to large shifts but not sensitive to small to moderate shifts. The EWMA chart is also a good alternative to the Shewhart control chart when we are interested in detecting small shifts. The performance of the EWMA control chart is approximately equivalent to that of the cumulative sum (CUSUM) control chart, and in some ways it is easier to set up and operate. To this end, Hong, Kang, Baek, and Kang (2008) proposed an EWMA-CV control chart in order to improve the Shewhart-type chart proposed by Kang et al. (2007) and detect small shifts more efficiently.

Castagliola, Celano, and Psarakis (2011) suggested a new method to monitor the CV by means of two one-sided EWMA charts of the CV squared. A numerical analysis demonstrated that this chart almost always performed better than the control chart proposed by Hong et al. (2008) even if this statistical outperformance is often rather small. However, the authors did not investigate the simultaneous monitoring of increasing or decreasing shifts in CV, which is important in real applications.

Recently, Calzada and Scariano (2013) developed a synthetic control chart for monitoring the CV. The results showed that the synthetic chart performed better than that of Kang et al. (2007), but worse than Castagliola et al. (2011) as long as the increasing shift in the CV is not too large. In addition, Castagliola et al. (2013a) evaluated an adaptive Shewhart control chart implementing variable sampling interval strategy to monitor the process CV. Castagliola et al. (2013b) proposed a Shewhart chart with supplementary run rules to monitor the CV. However, as they pointed out, the run rules charts for monitoring the CV does not outperform more advanced strategies like the chart proposed by Castagliola et al. (2011) or the synthetic chart proposed by Calzada and Scariano (2013).

The goal of this paper is to improve the performance of EWMA$_{-\gamma^2}$ chart based on the preliminary work of Castagliola et al. (2011) by proposing a new strategy for monitoring the coefficient of variation. The remainder of this paper is organized as follows. A brief review of the one-sided EWMA$_{-\gamma^2}$ chart of Castagliola et al. (2011) is given in Section 2. Following that, our modified EWMA chart is presented and the statistical performance of the new chart is investigated. Sets of optimal design parameters are also provided for different values of the in-control coefficient of variation, for different sample sizes, and for a wide range of deterministic shifts, including both decreasing and increasing cases in this section. The numerical comparisons with some other procedures are carried out in Section 3. The application of our proposed method is illustrated in Section 4 by a real data example from chemical process control. Several remarks conclude this paper in Section 5.

Now we summarize some abbreviated expressions used in this paper for easy reference and recapitulation.

- EWMA: Exponentially Weighted Moving Average; Cusum: cumulative sum.
- CV: coefficient of variation;
  MCV: modified chart for monitoring CV (this paper proposed);
  ECV: EWMA chart for monitoring CV (Castagliola et al., 2011);
  SynCV: synthetic chart for monitoring CV (Calzada & Scariano, 2013);
  SRCV: Shewhart chart with supplementary run rules for monitoring CV (Castagliola et al., 2013b);
- UCL: upper control limit; LCL: lower control limit; UWL: upper warning limit; LWL: lower warning limit;
- IC: in-control; OC: out-of-control;
- ARL: average run length; ZS-ARL: zero-state average run length; SDRL: standard deviation of the run length.

2. Monitoring CV with new modified EWMA chart

Suppose that we observe subgroups $X_{k1}, X_{k2}, \ldots, X_{kn}$ of size n at times $k = 1, 2, \ldots$. We also assume that there is independence within and between these subgroups and each random variable X_{kj} follows a normal $N(\mu_k, \sigma_k)$ distribution, where parameters μ_k and σ_k are constrained by the relation $\gamma_k = \frac{\mu_k}{\sigma_k} = \gamma_0$ when the process is in control. This implies that, from one subgroup to another, the values of μ_k and σ_k may change, but the coefficient of variation $\gamma_k = \frac{\mu_k}{\sigma_k}$ must be equal to some predefined in-control value γ_0, common to all the subgroups.

2.1. A brief review of EWMA$_{-\gamma^2}$ chart (Castagliola et al., 2011)

In this subsection, we give a brief review of the EWMA$_{-\gamma^2}$ chart proposed by Castagliola et al. (2011) (denoted as ECV chart). First, an upward ECV chart aims to detect an increase in the CV and is defined as

$$Z_k^+ = \max(\mu_0(\hat{\gamma}^2), (1-\lambda^+)Z_{k-1}^+ + \lambda^+\hat{\gamma}_k{}^2), \tag{1}$$

with $Z_0^+ = \mu_0(\hat{\gamma}^2)$ as the initial value and with the asymptotic corresponding upper control limit (UCL)

$$UCL = \mu_0(\hat{\gamma}^2) + K^+\sqrt{\frac{\lambda^+}{2-\lambda^+}\sigma_0(\hat{\gamma}^2)}. \tag{2}$$

Second, a downward ECV chart aims to detect a decrease in the CV and is defined as

$$Z_k^- = \min(\mu_0(\hat{\gamma}^2), (1-\lambda^-)Z_{k-1}^- + \lambda^-\hat{\gamma}_k{}^2), \tag{3}$$

with $Z_0^- = \mu_0(\hat{\gamma}^2)$ and with the asymptotic corresponding lower control limit (LCL)

$$LCL = \mu_0(\hat{\gamma}^2) + K^-\sqrt{\frac{\lambda^-}{2-\lambda^-}\sigma_0(\hat{\gamma}^2)}, \tag{4}$$

where $\mu_0(\hat{\sigma}^2)$ and $\sigma_0(\hat{\sigma}^2)$ are the mean and standard deviation of $\hat{\gamma}^2$ when the process is in control and $\lambda^+(\lambda^-)$ and $K^+(K^-)$ are the smoothing constant and chart coefficient of the upward (downward) ECV chart. Approximations for $\mu_0(\hat{\sigma}^2)$ and $\sigma_0(\hat{\sigma}^2)$ are provided by Breunig (2001) as

$$\mu_0(\hat{\gamma}^2) = \gamma_0^2\left(1 - \frac{3\gamma_0^2}{n}\right), \tag{5}$$

and

$$\sigma_0(\hat{\gamma}^2) = \left\{\gamma_0^4\left(\frac{2}{n-1} + \gamma_0^2\left(\frac{4}{n} + \frac{20}{n(n-1)} + \frac{75\gamma_0^2}{n^2}\right)\right) - \left(\mu_0(\hat{\sigma}^2) - \gamma_0^2\right)^2\right\}^{\frac{1}{2}}. \tag{6}$$

The suggested one-sided EWMA charts have many advantages according to Castagliola et al. (2011). However, it should be noted that, in Eq. (1), when $\mu_0(\hat{\gamma}^2) > (1-\lambda^+)Z_{k-1}^+ + \lambda^+\hat{\gamma}_k{}^2$, then $Z_k^+ = \mu_0(\hat{\gamma}^2)$. So, in the next time point, we have

$$Z_{k+1}^+ = \max(\mu_0(\hat{\gamma}^2), (1-\lambda^+)\mu_0(\hat{\gamma}^2) + \lambda^+\hat{\gamma}_{k+1}^2). \tag{7}$$

It is obvious that the samples collected before time $k+1$ are not used any longer. However, the advantage of the EWMA chart is that it will use not only the information of the current sample but also will use the former samples. To this end, in order to improve the performance of the ECV chart, next, we propose a modified EWMA chart based on the ECV chart. The comparison results showed that the new chart performs much better than the ECV chart, especially for detecting small to moderate shifts in CV.

2.2. Our modified methodology

To further enhance the sensitivity of the ECV chart in monitoring the process CV, we propose a modified procedure to the construction of Z_k^+ and Z_k^-. First, we define a new upward EWMA