Computers & Industrial Engineering 82 (2015) 33–40

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Alternative SBM model for Network DEA

Sebastián Lozano *

Dept. of Industrial Management, University of Seville, Spain

ARTICLE INFO

Article history:
Received 13 June 2014
Received in revised form 13 December 2014
Accepted 8 January 2015
Available online 14 January 2015

Keywords:
Efficiency
Network DEA
Slacks-based measure
SBM

ABSTRACT

In this paper a slacks-based measure (SBM) model for general networks of processes is presented. The proposed model differs from existing SBM Network Data Envelopment Analysis (DEA) approaches in that it considers the exogenous inputs and outputs at the system level instead of at the process level. It also relaxes the constraints for both the fixed-link and the free-link cases, thus enhancing the discriminating power of the model. To assess the performance of the individual processes an external efficiency model is presented. The proposed approach projects the system operation point onto the efficient frontier so that the target operation points of the different processes are externally efficient. The approach is illustrated with a problem from the literature.

1. Introduction

Data Envelopment Analysis (DEA) is a set of non-parametric techniques for assessing the relative efficiency of comparable (i.e. homogeneous) Decision Making Units (DMUs). Conventional DEA approaches consider the DMUs as black boxes that transform the inputs into outputs. There are, however, some DEA approaches, aptly labelled Network DEA (NDEA), that consider that the overall system is composed of different processes (a.k.a. subDMUs) with links between them that correspond to intermediate products that are produced and consumed internally within the system. These Network DEA approaches allow a more fine grained level of analysis and help to increase the discriminating power of the models. Some seminal Network DEA research corresponds to Färe and Grosskopf (1996, 2000) (see also Färe, Grosskopf, & Whittaker, 2007, chap. 12). However, it has been in the last few years that the interest in Network DEA has taken off, producing the relational Network DEA approach (Kao, 2009a, 2009b; Kao & Hwang, 2008, 2010), the Weighted Additive Efficiency Decomposition approach (Chen, Cook, Li, & Zhu, 2009; Cook, Zhu, Bi, & Yang, 2010), the SBM-NDEA approach (Tone & Tsutsui, 2009, 2010), the Network Slacks-Based Inefficiency (NSBI) approach (Fukuyama & Weber, 2010), the dynamic Network DEA approach (Tone & Tsutsui, 2014) and others (e.g. Fukuyama & Mirdehghan, 2012; Lozano, 2011; Maghbouli, Amirteimoori, & Kordrostami, 2014; Tavana, Mirzagoltabar, Mirhedayatian, Saen, & Azadi, 2013). The developments have been not only at the theory level but also in terms of a growing number of applications in different sectors, especially transportation (e.g. Lozano, Gutiérrez, & Moreno, 2013; Yu, 2010), supply chains (e.g. Agrell & Hatami-Marbini, 2013), finance (e.g. Avkiran, 2009; Ebrahimnejad, Tavana, Hosseinzadeh Lotfi, Shahverdi, & Yousefpour, 2014) and sports (e.g. Moreno & Lozano, 2014).

In this paper we present a new Network DEA model based on the well known SBM model (Tone, 2001). SBM uses a non-radial efficiency measure that in the non-oriented case is computed as a ratio of the average input reductions over the average output increases. SBM has some attractive features such as efficiency indication, monotonicity and unit invariance (Tone, 2001).

The structure of the paper is the following. In Section 2, after introducing appropriate notation, existing SBM Network DEA approaches are reviewed, analyzing their main features. In Section 3 the proposed Network SBM model for general networks of processes is formulated. Section 4 illustrates the proposed approaches using a problem from the literature. Finally, in Section 5 conclusions are drawn.

2. Existing Network SBM approaches

In this section the Network SBM approaches of Tone and Tsutsui (2009) and Yu (2010) are analyzed. First, the required notation and assumptions must be introduced. Thus, let us assume that there exist n DMUs all of which are structurally homogeneous, i.e. all of them have the same number of processes P and for all the DMUs the inputs and outputs of each process are the same. Let m and s be the total number of exogenous inputs consumed and of outputs produced, respectively. Let $I(p)$ be the set of exogenous inputs used

* Address: Escuela Superior de Ingenieros, Camino de los Descubrimientos, s/n, 41092 Sevilla, Spain. Tel.: +34 954487208; fax: +34 954487329.

E-mail address: slozano@us.es

http://dx.doi.org/10.1016/j.cie.2015.01.008
0360-8352/

in process p and, for each $i \in I(p)$, let x_{ij}^{p} denote the observed amount of exogenous input i consumed by process p of DMU j. Let $P_I(i)$ be the set of processes that consume the exogenous input i and $x_{ij} = \sum_{p \in P_I(i)} x_{ij}^{p}$ the total amount of exogenous input i consumed by all processes of DMU j. Similarly, let $O(p)$ be the set of final outputs of process p and, for each $k \in O(p)$, let y_{kj}^{p} denote the observed amount of final output k produced by process p of DMU j. Let $P_O(k)$ be the set of processes that produce the final output k and $y_{kj} = \sum_{p \in P_O(k)} y_{kj}^{p}$ the total amount of final output k produced by all processes of DMU j.

In addition to exogenous inputs and outputs, there exist R intermediate products. Let $P^{out}(r)$ the set of processes that generate the intermediate product r so that, for each $p \in P^{out}(r)$, let z_{rj}^{p} the observed amount of intermediate product r generated by process p of DMU j. Analogously, let $P^{in}(r)$ the set of processes that consume the intermediate product r and, for each $p \in P^{in}(r)$, let z_{rj}^{p} the observed amount of intermediate product r consumed by process p of DMU j. We assume that an intermediate product r cannot be consumed and produced simultaneously by a process, i.e. $P^{out}(r) \cap P^{in}(r) = \emptyset \quad \forall r$. Also, without loss of generality, let us assume that

$$\sum_{p \in P^{out}(r)} z_{rj}^{p} = \sum_{p \in P^{in}(r)} z_{rj}^{p} \quad \forall r \forall j \tag{1}$$

i.e. the intermediate products are completely generated and consumed within the own DMU. Finally, to facilitate the model formulation, it is convenient to define the sets $R^{out}(p)$ and $R^{in}(p)$ corresponding to the intermediate products produced and consumed, respectively, by a certain process p.

Note that the sets $P^{out}(r)$ and $P^{in}(r)$ (and, equivalently, $R^{out}(p)$ and $R^{in}(p)$) jointly determine all the structure of intermediate flows within the system. Thus, for example, a system consisting of just parallel process with no intermediate flows ($R = 0$) would have $R^{out}(p) = R^{in}(p) = \emptyset \quad \forall p$. On the contrary, a typical multi-stage series system would have $R^{out}(p) = R^{in}(p+1) \quad 1 < p < P$ and $R^{in}(1) = R^{out}(P) = \varnothing$.

Before proceeding further, it must be taken into account the alternative way in which intermediate products can be considered (e.g. Avkiran, 2009; Tone & Tsutsui, 2009). In those papers the intermediate flows are modelled as directed links and each link between two processes is, in principle, a different intermediate product. On the contrary, we allow each intermediate product to be produced by more than one process in which case those productions are pooled and from that pool the intermediate product is extracted to be consumed by possibly multiple processes. Therefore, except in the case $|P^{out}(r)| = |P^{in}(r)| = 1$ there are no directed links between the processes. In other words, except in the case that the intermediate product is produced by just one process and consumed by just one other process, there is no information about, and it does not matter, which specific process produced the intermediate product units consumed by a certain process.

As for the decision variables, and using J for the DMU to be assessed, let

ξ_J	Overall efficiency of DMU J as per model (2)–(6)
λ_j^{p}	Intensity variable of process p of DMU j
s_i^{p-}	Potential reduction (i.e. slack) of input i of process p
s_k^{p+}	Potential expansion (i.e. shortfall) of output k of process p

With the above notation and assumptions and for a general network of processes, the SBM Network DEA (SBM-NDEA) model of Tone and Tsutsui (2009) can be formulated as:

SBM-NDEA model

$$\xi_J = \text{Min} \frac{\sum_p w_p \cdot \left(1 - \frac{1}{|I(p)|} \sum_{i \in I(p)} \frac{s_i^{p-}}{x_{iJ}^{p}}\right)}{\sum_p w_p \cdot \left(1 + \frac{1}{|O(p)|} \sum_{k \in O(p)} \frac{s_k^{p+}}{y_{kJ}^{p}}\right)} \tag{2}$$

subject to

$$\sum_j \lambda_j^{p} x_{ij}^{p} = x_{iJ}^{p} - s_i^{p-} \quad \forall i \forall p \tag{3}$$

$$\sum_j \lambda_j^{p} y_{kj}^{p} = y_{kJ}^{p} + s_k^{p+} \quad \forall k \forall p \tag{4}$$

$$\sum_{p \in P^{out}(r)} \sum_j \lambda_j^{p} z_{rj}^{p} = \sum_{p \in P^{in}(r)} \sum_j \lambda_j^{p} z_{rj}^{p} \quad \forall r \tag{5}$$

$$\lambda_j^{p} \geqslant 0 \quad \forall j \forall p \quad s_i^{p-} \geqslant 0 \quad \forall i \forall p \quad s_k^{p+} \geqslant 0 \quad \forall k \forall p \tag{6}$$

where w_p are normalized weights representing the importance of process p (division p in the terminology of Tone & Tsutsui, 2009).

Note that the above formulation corresponds to the Constant Returns to Scale (CRS) case. The objective function (2) represents the ratio of the weighted average input reduction of the different processes to the weighted average output expansion of the different processes. The numerator is less than or equal to one while the denominator is greater than or equal to one. As regards the constraints, (3) compute the potential input reduction for each process. Note that the non-negativity of the slack variables s_i^{p-} means that no increase is considered in any of the processes. Similarly, (4) measures the potential output increase for each process. The non-negativity of the shortfall variables s_k^{p+} forbids that any process reduces any output.

Finally, constraints (5) represent global balance equations for the intermediate products, i.e. the amount consumed by the different process must match the amount produced. This corresponds to the so-called free-links case (Tone & Tsutsui, 2009). The fixed-links case would substitute constraints (5) by

$$\sum_j \lambda_j^{p} z_{rj}^{p} = z_{rJ}^{p} \quad \forall r \forall p \in P^{out}(r) \cup P^{in}(r) \tag{7}$$

Note also that model (2)–(6) is non-oriented. The input-oriented and the output-oriented cases would change the objective function to

$$\xi_J = \text{Min} \sum_p w_p \cdot \left(1 - \frac{1}{|I(p)|} \sum_{i \in I(p)} \frac{s_i^{p-}}{x_{iJ}^{p}}\right) \tag{8}$$

and

$$(\xi_J)^{-1} = \text{Max} \sum_p w_p \cdot \left(1 + \frac{1}{|O(p)|} \sum_{k \in O(p)} \frac{s_k^{p+}}{y_{kJ}^{p}}\right) \tag{9}$$

respectively.

The basic feature of the SBM-NDEA is that the input and output changes are computed at the process level and relative to the input and outputs of that process. Another feature is that the relative input reductions of the different processes are aggregated before dividing them by the aggregated output increases of the different processes. Finally, as mentioned above, the aggregation involves different weights for the different processes.

One of the advantages of the SBM-NDEA is that it allows the computation of efficiency scores for each process and, in the input