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The consignment stock of inventories in coordinated model with generalized policy ☆

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ABSTRACT

In this paper, coordination between a single vendor (or manufacturer) and a buyer (or retailer) via the delivery schedule in a production and distribution system is presented. A continuous deterministic model with centralized decision process is developed. To satisfy the buyer's demands, the product is delivered in discrete batches from the vendor's stock to the buyer's stock and all shipments are realized instantaneously. A more general type of consignment stock (CS) policies for the vendor–buyer integrated production–distribution model is analyzed. Our model does not require equal in size shipments. The inventory patterns and the cost structure of production distribution cycles (PDC) are described in different scenarios. A comparative study of the results shows that the generalized CS policies perform better. Considering CS-policies Braglia and Zavanella (2003) ask about a possibility of cost reduction by delaying a number of late deliveries. Unfortunately, a negative answer was given by Zanoni and Grubbström (2004). We verify this problem to obtain a positive answer in more general setting. A solution procedure is developed to find optimal generalized CS-policy for the problems with nonequal and equal in size deliveries. Optimal solutions are found and illustrated with numerical examples.

1. Introduction

Recently coordinated behaviors of the members of the supply chain are considered as important. A well integrated supply chain involves coordination of the flows of materials and information between distinct entities (as supplier, manufacturer, transporter, buyer, etc.). Over the years various integrated inventory models have been developed to determine feasible (conditional) optimal control policies. A coordinated management can reduce average holding inventory level and the total system cost per unit time.

The idea of joint optimization for vendor and buyer was initiated by Goyal (1976) and Banerjee (1986). A common assumption that has been made is that the product is delivered in discrete batches from the vendor's stock to the buyer's stock and that the stock holding cost increases as the inventory moves down the supply chain. For such a case Lu (1995) described optimal production distribution cycle under the assumption that all shipments are equal in size and delivered just in time for the buyer.

Other authors incorporated policies in which the sizes of successive shipments (from the vendor to the buyer) either are increased by a factor or are equal in size. Hill (1999), combining these two type of policies, derived the structure of a globally optimal production distribution cycle (PDC). The assumption that holding cost at the vendor is not greater than the holding cost at the buyer was relaxed in later research (see Braglia & Zavanella (2003) and Hill & Omar (2006)). For a more recent and comprehensive review of the progress of supply chain management research, see Sarmah, Acharya, and Goyal (2006) and Glock (2012).

The basic consignment stock approach requires the deliveries to be made as soon as sufficient stock has been produced by the vendor with a constraint on maximal buyer's inventory positions. Braglia and Zavanella (2003) are probably the first to have studied such kind of an inventory system. Considering CS-policies they ask about a possibility of cost reduction by delaying a number of late deliveries. A negative answer was given by Zanoni and Grubbström (2004). We verify this problem to obtain a positive answer in more general setting.

While globally optimal production distribution cycle (PDC) has been obtained by policies with non equal shipments, CS policies are still attractive. There are various models that describe problems of coordinating distribution systems under vendor-managed inventory with consignment arrangements (Gümüs, Jewkes, & Bookbinder, 2008; Chen, Lin, & Cheng, 2010; Ru & Wang, 2010; Hariga, Gümüs, Daghfous, & Goyal, 2013). In CS models, the considered shipment comprise two type of deliveries, i.e. just-in-time

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(JIT) or delayed for the vendor. Unlike the hitherto existing CS-models, the model presented in this paper neither requires equal in size of all deliveries nor assumes the delivery (transportation and replenishment) cost per shipment is the same for JIT and for delayed deliveries.

Many contemporary researchers base their study on Braglia and Zavanella deterministic model. In this context, Zanoni, Jabar, and Zavanella (2012) investigate and compare different policies (equal or nearly equal in size) when the vendor's production process is subject to learning effects. For multi-buyers case it has been defined in Zavanella and Zanoni (2009). Yi and Sarker (2013) extend the study for policy with controlled lead time and buyer's space limitation. Such policies are considered not only in models with centralized decisions and deterministic setting. The results in Tang, Zanoni, and Zavanella (2007) indicate, that the CS policy could be a strategic and profitable approach to improve supply chain performance in uncertain environments.

In Bylka (2013) the assumption on equal in size deliveries has been relaxed. In general, a supply chain is composed of independent partners with individual preferences and costs. When applied to productive environments, it allows (for isolated situations) the vendor to calculate the Economic Production Quantity (EPQ). It may be significantly different from the buyer's EOQ. Generalized consignment stock policies (as non-cooperative strategies) admit two sizes of shipments taking into consideration vendor's and buyer's preferences respectively. It is natural, that potential savings in cooperation (in centralized case) cannot be ignored. One of the purpose of our research is to examine the generalized CS-policies for integrated two stage supply chain. Final remark is that, in comparison with the most recent literature on CS policies the sizes of deliveries as well as their delivery costs (i.e. ordering and transportation costs for each shipment) are different for JIT and for delayed deliveries.

The paper is organized as follows. In Section 2, we develop the model describing inventory patterns under (k,n)-consignment stock policies (CS(k,n)-policies). Conditions for the costs of generalized CS policies are presented in Section 3. Optimal α-proportional CS-policies are given in Sections 3 and 4. Some properties of generalized CS policies with well proportional sizes of JIT and delayed deliveries are presented in Section 5.

2. Vendor–buyer relationships under CS-policies

We consider a continuous deterministic model of a production–distribution system for a single product. Our objective is to develop an economic lot size model to minimize the integrated supply chain cost. First we list the notation of the parameters used throughout the paper. Let $i = 0$ be the index for the vendor and $i = 1$ be the index for the buyer.

T	the length of PDC (time)
Q	the production lot size per PDC (quantity)
A	the fixed set up costs of production, ($)
A_i	the delivery (transportation and replenishment) cost per shipment ($)
h_i	the unit stock holding cost for the vendor ($/quantity $\times$ time)
P	the production rate per unite of time (quantity/time)
D	the demand rate of the buyer (quantity/time), the ratio $\lambda = \frac{P}{D}$

We assume:

[1] Constant production rate is sufficient to meet buyer's demand ($\lambda > 1$) and buyer's demand must be satisfied.

[2] The final product is distributed by shipping it in discrete lots from the vendor's stock to buyer's stock (realized instantaneously).

[3] The time horizon is infinite and a schedule is determined by a sequence of production distribution cycles (PDC)

- In PDC schedule, the buyer receives deliveries in two phases – just in time (directly after their completion) or possible delayed for the vendor. It will be defined formally after introducing notations.

Each PDC is determined by the size of production batch Q, the number of deliveries $m > 0$ and a **schedule of deliveries**. By the assumption [3], the objective is to determine a PDC schedule which minimizes the average joint total cost of production, shipment and stockholding.

Then, for a PDC on the time interval $[0, T]$, we introduce the notation:

m	the number of successive shipments (deliveries) to the buyer
q_j	the quantities of successive shipments, $j = 1, \ldots, m$
t_j	the timing of successive shipments (time), $j = 1, \ldots, m$ $[(q_1, t_1), \ldots, (q_m, t_m)]$ the delivery schedule, $\sum_{j=1}^{m} q_j = Q$
q_0	the initial inventory position $q_0 = I^b(0)$ at the buyer's stock
$I^v(t)$	the vendor's inventory position at t (quantity)
$I^b(t)$	the buyer's inventory position at t (quantity)
$I^b(t^+)$	the buyer's inventory at t just after the possible delivery
$\tilde{I}^v$, $\tilde{I}^b$	the individual total stock holding value (quantity $\times$ time)
I^b_{max}	the maximum level of buyer's inventory position (quantity)
(q^v, k)	the size (quantity) and number of initial deliveries
(q^b, n)	the size (quantity) and number of final deliveries
$\alpha = \frac{q^b}{q^v}$	the ratio of sizes of deliveries
(q, k, n)	α-proportional CS policy, $q^v = q,\ q^b = \alpha q$

In this paper we impose the following assumption on PDC:

[4] In PDC schedule, the buyer receives $m = k + n$ deliveries under the following specific conditions (individual agents' preferences):

(1) The initial $k \geqslant 0$ deliveries equal in size q^v are sent <u>as soon as</u> it is possible.

(2) The final $n > 0$ deliveries are equal in size $q^b = \alpha q^v$ and the buyer's inventory level reaches its maximum I^b_{max} after each replenishment.
For convenience, we assume that $k = 0$ implies $\alpha = 1$ (an equal in size deliveries case).

Under this assumption, the initial k deliveries each of the size q^v (as the vendor preferred) precede n deliveries each of the size q^b (on buyer's mode). See also Bylka (2013) for the non-cooperative case. In the initial phase the vendor's stock becomes empty after each of k or $k + 1$ initial deliveries. In the second phase the last n or $n - 1$ deliveries can be delayed. It is important to notice that a consignment stock agreement ought to guarantee that the quantity stored in the buyer's warehouse is as high as it is possible under the constraint that the inventory position is not greater than the maximum level I^b_{max}. Two possibilities will be considered: to set I^b_{max} as low (Scenario 1) or as high (Scenario 2) as possible. It can be specified depending on the cases $h_0 < h_1$ or $h_0 > h_1$.