



A new hybrid algorithm for the balanced transportation problem[☆]



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ABSTRACT

We propose a heuristic-exact hybrid algorithm that consists of a heuristic phase, based on two novel heuristics, followed by an exact phase, based on an adapted Ford–Fulkerson algorithm, to solve the Balanced Transportation Problem (BTP). First, we propose three alternative primal models for the BTP. We also define the concepts of negative sets, negative dual rectangles, and the optimal tableau for the BTP. Next, we explore the relationships between these concepts. We also propose two greedy heuristics, based on a linear programming relaxation of the BTP model, to find some negative sets and negative dual rectangles. These two heuristics turn out to be very efficient and obtain optimal or near-optimal BTP tableaus rapidly, as confirmed by the computational experiments. Then, an adapted Ford–Fulkerson algorithm is presented and used to find an optimal solution. The two important advantages of our adapted Ford–Fulkerson algorithm over the standard Ford–Fulkerson algorithm are more flexibility and efficiency. Extensive computational results show that the growth in run-time of our hybrid algorithm, on average, is approximately a linear function of the BTP size. It has significant advantage over the transportation simplex method and on the largest problem instances it is almost five times faster. A key feature of the proposed algorithm is that it is free of degeneracy and cycling altogether.

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1. Introduction

The transportation problem (TP) is a special type of linear program which seeks to find the minimum cost way to transport goods from a set of sources or supply locations $i = 1, \dots, m$ to a set of destinations or customers $j = 1, \dots, n$. Supply location i has a supply of s_i units, and customer j has a demand of d_j units. The cost per unit transported from supply location i to customer j is denoted by c_{ij} , and the number of units transported is denoted by x_{ij} . If the sum of the supplies at the sources equals the sum of the demands at the destinations, then the problem is called a Balanced Transportation Problem (BTP). Companies that have multiple manufacturing sites, many distribution centers and warehouses, and multiple customers, use the transportation problem to minimize their distribution costs while satisfying all the constraints. The transportation problem also has applications in *signature matching*, where one signature could be defined as the source and the other as the customer and by setting the cost for a source-customer pair to the ground distance between an element

in the first signature and an element in the second (Rubner, Tomasi, & Guibas, 2000). The transportation problem is a special case of the *minimum cost flow problem* where all the sources are on one side of a bipartite graph and all the destinations on the other side. A number of applications of the minimum cost flow problem, including multiperiod production and inventory planning problems and cash management problems, are discussed in Glover, Klingman, and Phillips (1992) and Ahuja, Magnanti, and Orlin (1993).

In this work, we consider three primal models for the BTP and define the concepts of negative sets and negative dual rectangles. We also propose two greedy heuristics based on a linear programming relaxation of the BTP model to rapidly find a near-optimal BTP tableau. Then, an adapted Ford–Fulkerson algorithm is presented and used to find an optimal solution. The two important advantages of our adapted Ford–Fulkerson algorithm over the standard Ford–Fulkerson algorithm are flexibility and efficiency. Finally, we propose a hybrid algorithm that consists of these two heuristics and the adapted Ford–Fulkerson algorithm to solve the BTP. Extensive computational results show that our hybrid algorithm has a performance advantage over the transportation simplex method and GAMS/CPLEX 12 software on large problem instances since this hybrid algorithm is free of degeneracy and cycling altogether.

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In brief, the key contributions of this work are:

- We propose a hybrid algorithm completely free of degeneracy and cycling.
- The hybrid algorithm is more efficient than the transportation simplex method.
- The algorithm's run-time grows, nearly, as a linear function of problem size.
- We propose two very efficient heuristics for the BTP.
- We present an improved Ford–Fulkerson algorithm for the BTP.

The rest of the paper is organized as follows. In the next section, a brief literature review of the TP is presented. In Section 3, we provide some preliminaries and basic assumptions. In Section 4, the hybrid algorithm is described in detail. Computational Experiments are reported in Section 5. Section 5 also includes computational comparison of the proposed algorithm with the transportation simplex method and the GAMS/CPLEX 12 software. Finally, Section 6 concludes the paper and presents further research directions.

2. Literature review

The first statement of the classical TP is due to Hitchcock in a 1941 paper in which he also sketched out a solution procedure (Hitchcock, 1941). In (Koopmans, 1949) Koopmans independently investigated and solved the same problem. He also demonstrated the relationship between basic solutions in the TP and the tree structure of a graph, thus the problem is referred to as the Hitchcock–Koopmans transportation problem. The simplex method for the linear programming problem was developed by Dantzig (1951a). Then, he adapted the simplex method to solve the transportation problem (Dantzig, 1951b). Munkres extended the Hungarian method to solve the TP (Munkres, 1957). He introduced a method to find the minimum covering lines in the assignment problem and extended it to solve the TP. Ford and Fulkerson (1957) introduced a method to solve the maximal network flow problem which then they used to solve the TP. Arsham and Khan (1989) proposed an alternate method to stepping-stone for solving the TP. The auction algorithm for assignment problem (Bertsekas, 1979, 1988) was generalized to solve the TP by Bertsekas and Castanon (1989). Orlin (1993) described a strongly polynomial time algorithm for the general uncapacitated minimum cost flow problem that can be exploited to solve the TP. By casting the TP as a slightly different form, Sharma and Sharma (2000) obtained a dual problem that has a special structure. They exploited that structure and introduced a heuristic to generate a good starting solution for dual based approaches that are used for solving the TP. Sharma and Prasad (2003) introduced a heuristic that uses the dual solution obtained by Sharma and Sharma (2000) and gives a good starting solution for the primal transportation problem. Papamantou, Paparrizos, and Samaras (2004) presented an experimental computational study to compare the classical primal simplex algorithm and the exterior point algorithm for the TP. In 2008, Brenner (2008) presented a new algorithm for the TP with a worst-case run-time of $O(mn^2(\log m + n \log n))$. Efficient implementations of minimum-cost flow algorithms are presented in Kiraly and Kovacs (2012). A comprehensive study of network flow theory and algorithms can be found in, for example (Korte & Vygen, 2012). A recent survey related to minimum-cost network flow is presented in Sifaleras (2013). A survey of the many variants of transportation problem can be found in Díaz-Parra, Ruiz-Vanoye, Loranca, Fuentes-Penna, and Barrera-Cámara (2014). Kovács reports that, in general, the primal network simplex and cost-scaling are the most efficient and robust algorithms to solve minimum

cost flow problems. Other methods, though, can outperform them in particular cases (Kovács, 2014).

Our work aims at developing a new hybrid algorithm, to solve the BTP, by introducing two greedy heuristics based on a linear programming relaxation of the BTP model and improving the standard Ford–Fulkerson method (Dantzig, 1951a). To that end, we first present three equivalent linear programming models for the BTP and define the concepts of negative sets, negative dual rectangles, and the optimal tableau for the BTP. Then, we explore the relationships between these concepts and propose two heuristics to find some negative sets and negative dual rectangles. Finally, we present and compare the proposed hybrid algorithm with the transportation simplex method on 1800 randomly generated instances with up to 3000 sources and destinations.

3. Preliminaries

In this paper, we assume that the BTP cost matrix is a non-negative matrix, i.e., $C \geq 0$. In this section, we present three equivalent linear programming models for the BTP and prove five propositions and three corollaries. Then, we present the definitions of the First and the Second Type Negative Dual Rectangles followed by the definitions of the First and the Second Type sets.

Three equivalent linear programming models for the BTP are presented in Fig. 1 as BTP-1, BTP-2, and BTP-3 models. It is not difficult to prove the equivalency of these models.

Proposition 1. A BTP tableau is optimal if there is a feasible solution $X = [x_{ij}]_{m \times n}$ such that $c_{ij} \geq 0$ and $c_{ij} \times x_{ij} = 0$ for all i, j .

Proof. The proposition is immediate by noting that the objective value is zero at such a feasible solution. $\square$

Proposition 2. A BTP tableau is optimal if and only if its dual objective function value is zero.

Proof. This follows from Proposition 1 and the strong duality theorem of linear programming. $\square$

Proposition 3. If a constant is added to each cost in a row (or column) of a BTP tableau, then the optimal solution to the problem is unchanged.

Proof. The proposition is followed immediately by considering Model BTP-3 (Fig. 1). $\square$

Now, suppose that the BTP is formulated as Model BTP-1 (Fig. 1). Then its dual model, DBTP-1, is given in (1).

Model DBTP-1 :

$$\text{Max } w = \sum_{i=1}^m s_i u_i - \sum_{j=1}^n d_j v_j \quad (1)$$

s.t.

$$\begin{aligned} u_i - v_j &\leq c_{ij} \\ u_i &\geq 0 \quad (i = 1, \dots, m) \\ v_j &\geq 0 \quad (j = 1, \dots, n) \end{aligned}$$

Thus, in the remainder of the paper, we assume that (u, v) are the dual variables.

Proposition 4. If a BTP tableau is not optimal, then there must be nonnegative numbers u_i^* and v_j^* such that:

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