Computers & Industrial Engineering 82 (2015) 159–166

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

A scheduling model with a more general function of learning effects ☆

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ARTICLE INFO

Article history:
Received 13 July 2014
Received in revised form 22 January 2015
Accepted 25 January 2015
Available online 7 February 2015

Keywords:
Scheduling
Learning effect
Single-criterion
Multi-criteria

ABSTRACT

This study develops a more general model for scheduling problems with learning effects. Compared with the existent general models, the proposed time- and position-dependent model simultaneously covers the normal and actual processing cases. Moreover, the model has many new properties that the previous work did not study. In this paper, a distinctive proof technique is developed based on the adding-and-subtracting-terms operation and the Lagrange Mean Value Theorem. The proof technique is easier to use than the method based on multiple identical or similar lemmas employed in a large number of literatures.

1. Introduction

Job scheduling and learning effects have respectively received considerable attention from two different communities since the early of the last century. However, it is surprising that the scheduling problem with learning effects has not been investigated until the beginning of this century (Biskup, 1999; Cheng & Wang, 2000). Many models since then have arisen to formulate the role of learning effects in short term production planning. These models will be reviewed in the next section.

To further unify previous works, we developed a more general model, $p_{ir}^{\mathrm{A}} = p_i^{\mathrm{N}} f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}^{\mathrm{A|N}}, r\right)$, where superscript A and N respectively represent the actual and the normal processing case. *The proposed model almost covers existent models with learning effects. Moreover, we discover some new findings based on the model.* For example, many common rules are no longer effective for the models based on the sum of the actual processing time of the jobs already processed, such as the Shortest Processing Time (SPT) rule, the Weighted Shortest Processing Time (WSPT) rule, the Earliest Due Date (EDD) rule and the Weighted Earliest Due Date (WEDD) rule. As a result, the $p_{ir}^{\mathrm{A}} = p_i^{\mathrm{N}} f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}^{\mathrm{A}}, r\right)$ model has no longer the polynomial solvable properties of the $p_{ir}^{\mathrm{A}} = p_i^{\mathrm{N}} f\left(\sum_{k=1}^{r-1} \beta_k p_{[k]}^{\mathrm{N}}, r\right)$ model. In multi-criteria scheduling, the Panwalkar's transformation is effective for the common due-date assignment problem with the general position-dependent learning effect, but the Panwalkar's scheduling rule is no longer effective.

Another highlight of this study is to present a distinctive proof technique. The majority of the previous works employed multiple identical or similar lemmas to prove the properties of their models (Kuo & Yang, 2006; Yang & Kuo, 2007; Cheng, Wu, & Lee, 2008; Wang, 2008; Wu & Lee, 2008; Yin, Xu, Sun, & Li, 2009; Cheng, Lai, Wu, & Lee, 2009; Yin, Xu, & Wang, 2010; Lai & Lee, 2011; Lee & Lai, 2011; Wang & Wang, 2011; Yin, Liu, Hao, & Zhou, 2012; Low & Lin, 2013; Wang & Wang, 2013; Lu & Wang, 2013; Yang, Cheng, & Kuo, 2013). It is easy to understand those lemmas, but the developments are not as straightforward as they seem because it is difficult to construct the auxiliary functions for these lemmas. As shown in the papers employed these lemmas, we have to make some transformations to find extra coefficients α, λ, λ_1, and λ_2. Nevertheless, the proposed technique is based on the adding-and-subtracting-terms operation and the Lagrange Mean Value Theorem. The adding-and-subtracting-terms operation is common in mathematics. The Lagrange mean value theorem is a fundamental theorem in calculus. The usage of the proposed technique does not require extra coefficients and functions. Therefore, the proposed technique is simpler and easier to use.

2. Related literature review

In this section, we will review existent scheduling models with learning effects. For more comprehensive reviews, the reader refers to the work of Biskup (2008).

Existent related models utilize two approaches to characterizing the role of learning effects in the short-term production planning. One is based on the position of the job being processed and another is based on the sum of the processing time of all jobs processed.

☆ This manuscript was processed by Area Editor T.C. Edwin Cheng.
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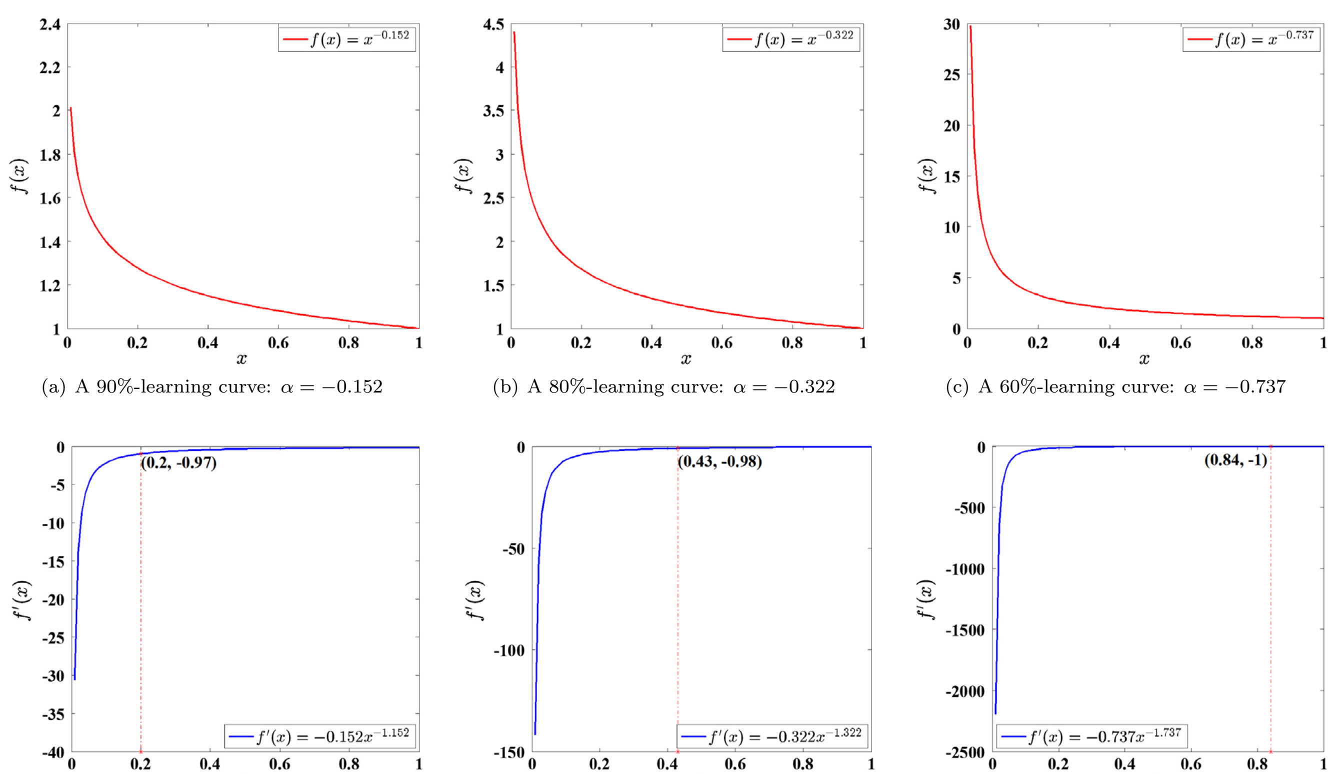
http://dx.doi.org/10.1016/j.cie.2015.01.025
0360-8352/

The position-based learning models have a common assumption that the learning occurs as a result of repeating processing-time-independent operations, like setting up the machines. This assumption is in accord with the case that the processing of jobs has hardly human interference, such as, the processing of memory chips and circuit boards. The first position-based model developed by Biskup (1999) formulates autonomous learning effects in scheduling environment by $p_{ir} = p_i r^{\alpha}$, where p_i is the normal processing time and p_{ir} is the actual processing time of the i-th job if it is scheduled in the r-th position; α denotes a non-positive learning index. This concise formulation describes the learning by repeating processing-time–independent tasks, such as setups, reading data, etc. Lee and Wu (2004) expanded the unit cost learning model into two-machine flow shop scheduling under the assumption that learning takes place on each machine separately. But, Mosheiov and Sidney (2003) argued that the learning processing may be significantly affected by the job itself, and then proposed an extension version in which learning rates differ from job to job, namely, $p_{ir} = p_i r^{\alpha_i}$. Cheng and Wang (2000) approximately described the learning effect by a piecewise linear decreasing function, $p_{ir} = p_i - v_i \min(r-1, n_{0i})$, where v_i is a learning effect coefficient and n_{0i} is a threshold at which the learning curve will plateau, moreover, $v_i n_{0i} < p_i$. Bachman and Janiak (2004) suggested a simpler formulation, $p_{ir} = p_i - v_i r$. Then, Wang and Xia (2005) and Xu, Sun, and Gong (2008) expanded the simple linear model into multi-machine scheduling under the assumption that each job has an identical learning effect. However, the processing time of the jobs already produced is completely neglected in these position-based models. If human interaction have a significant impact on scheduling, the workers' experiences will affect the processing time of each job.

The time-based learning models characterize the experiences the workers gain from producing the jobs. As Biskup (2008) has mentioned, offset printing, airplane maintenance and high-end electric production are examples for the approach. Kuo and Yang (2006) proposed a time-dependent model, $p_{ir} = p_i\left(1 + \sum_{k=1}^{r-1} p_{[k]}\right)^{\alpha}$, where $\alpha \leqslant 0$ is a constant learning index; $p_{[k]}$ denotes the normal processing time of a job if it is scheduled in the k-th position. Koulamas and Kyparisis (2007) proposed another variant, $p_{ir} = p_i\left(1 - \sum_{k=1}^{r-1} p_{[k]} \Big/ \sum_{k=1}^{n} p_{[k]}\right)^{\alpha}$, where $\alpha \geqslant 1$. But, it is questionable that the learning effect in this model strongly depends on the normal processing time of unprocessed jobs. Cheng et al. (2009) pointed out that the actual processing time of a given job drops to zero precipitously as the number of jobs increases in the position-based models and when the normal job processing time is large in the sum-of-processing-time-based models. Motivated by the observation, they developed a new time-based model, $p_{ir} = p_i\left(1 + \sum_{k=1}^{r-1} \ln p_{[k]}\right)^{\alpha}$, where $\alpha \leqslant 0$. The model with a logarithm function has an advantage that it is subject to the law of learning rate diminishing return.

Both position- and time-dependent models have their validity. The former is more suitable for a fully automatic environment while the latter is more appropriate for a human interference environment. Undoubtedly, the learning effects of machines and humans may simultaneously exist in some situations, such as, robots with neural networks in the assembly line. A robot modifies its actions through self-learning in processing jobs. Meanwhile, the operators in the control center learn how to give the commands efficiently through working experience. Hence, Cheng et al. (2008) developed an aggregate model, $p_{ir} = p_i\left(1 - \sum_{k=1}^{r-1} p_{[k]} \Big/ \sum_{k=1}^{n} p_{[k]}\right)^{\alpha_1} r^{\alpha_2}$, where, $\alpha_1 \geqslant 1$ and $\alpha_2 < 0$ respectively denote the sum-of-processing-time-based learning index and the job-position-based learning index. Yin et al. (2009) proposed a general model, $p_{ir} = p_i f\left(\sum_{k=1}^{r-1} p_{[k]}\right) g(r)$, with a differentiable non-increasing



(a) A 90%-learning curve: $\alpha = -0.152$

(b) A 80%-learning curve: $\alpha = -0.322$

(c) A 60%-learning curve: $\alpha = -0.737$

(d) The first order derivative of the 90%-learning function

(e) The first order derivative of the 80%-learning function

(f) The first order derivative of the 60%-learning function

Fig. 1. Three exponential learning curves and the corresponding first order derivatives.