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One for one period policy for perishable inventory ☆

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ABSTRACT

Recently, for zero ordering cost a new ordering policy named $(1, T)$, in which the time interval between two consecutive orders and the value of the order size are both constant, have been developed for nonperishable products. In this paper, the $(1, T)$ policy is developed for perishable products. Using an analogy among this inventory model, a queueing model with impatient customers, and a finite dam model, the long-run average total cost function of the inventory system is derived. It is observed that the total cost rate is independent from the lead time as is for nonperishable products. Since analyzing the convexity of the model is extremely complicated, a proposition is proved to define a domain for the optimal solution, and then a search algorithm is presented to obtain the optimal solution. Furthermore, a numerical analysis is carried out to examine the sensitivity of optimal T with respect to system parameters and to compare the performance of $(1, T)$ policy with the well known $(S-1, S)$ policy. This analysis shows that for fixed values of system parameters, there is a fixed value of lead time for which the performance of $(1, T)$ policy is better than $(S-1, S)$ policy. Further as the lead time increases this superiority is more pronounced.

1. Introduction

Considering nonperishable products, Haji and Haji (2007) introduce a new inventory control policy called $(1, T)$, which is different from the classical inventory policy used in the literature of inventory and production control systems. The $(1, T)$ policy is to order one unit at each fixed time period T. For the case of stochastic demand, the $(1, T)$ inventory policy is the one in which the time interval between two consecutive orders and the value of the order size are both constant. Therefore, when $(1, T)$ is employed in the first level of a supply chain, it prevents expanding the demand uncertainty for other levels and makes their demand deterministic, one unit every T units of time. Hence, the following advantages are obtained using the $(1, T)$ policy in retailers' level of a supply chain (Haji and Haji (2007)): (1) The safety stock in supplier is eliminated (cost reduction). (2) Shortage cost in supplier due to uncertainty in demand is eliminated. (3) Information exchange cost for supplier due to the elimination of uncertainty of its demand is eliminated. (4) Inventory control and production planning in supplier are simplified. And (5) This policy is very easy to apply. Following Haji and Haji (2007), Haji, Pirayesh Neghab, and Baboli (2009) apply the $(1, T)$ policy to a two-echelon inventory system with nonperishable products. In this paper, we consider the $(1, T)$ policy for perishable products in a single stage inventory system.

The assumption of infinite lifetime is common in most of the inventory models. However, the perishability of products is a major problem of some industry sectors in which by disregarding the finite lifetime of their products the resulting model may give inaccurate results. Hence, significant studies have been carried out for inventory control of perishable products. Nahmias (1982) and Karaesman, Scheller-Wolf, and Deniz (2011) present two comprehensive reviews focusing on inventory control of perishable products.

Schmidt and Nahmias (1985) for perishable items consider a system operating under $(S-1, S)$ policy with lost sales, Poisson demand, outdating costs, purchase costs, and per unit per period holding costs. They derive the cost function by solving partial differential equations for the S-dimensional stochastic process corresponding to the time elapsed since the last S orders were placed. Perry and Posner (1998) generalize Schmidt and Nahmias (1985) to allow for general types of customer impatience behavior. Olsson and Tydesjo (2010) extend Schmidt and Nahmias (1985) by allowing backorders.

In this study, we consider an inventory system under Schmidt and Nahmias (1985) assumptions but employ the $(1, T)$ policy instead of the $(S-1, S)$ policy. Using some concepts from queueing theory, it is shown that the considered model is similar to a $D/M/1$ queue with impatient customer. The long-run average total cost function including the total outdating, holding, shortage and

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purchase costs is derived utilizing the analogy of the $D/M/1$ queue with impatient customer and the finite dam model. Since analyzing the convexity of the model is extremely complicated, a proposition is presented to introduce a domain for optimal T. Therefore, the optimal solution can be obtained using a search algorithm.

Haji and Haji (2007) prove the independency of the total cost function of $(1, T)$ policy from the lead time for nonperishable products. In this study, we show that this result is also valid for the case of perishable products. Further, for the case of perishable products we have compared $(1, T)$ and $(S-1, S)$ policies. The results show that for fixed values of system parameters, there is a fixed value of lead time for which the performance of $(1, T)$ policy is better than $(S-1, S)$ policy. Furthermore, thorough a numerical example we show as the lead time increases this superiority is more pronounced.

The proceeding parts of this paper are organized as follows: In Section 2, the model and its analogy to other models are described. In Section 3, the cost function of the model is derived and an algorithm is presented to obtain the optimal solution. In Section 4, a numerical analysis is carried out. Finally in Section 5, the conclusion is presented.

2. Problem description and mathematical formulation

2.1. Problem description

An inventory system with lost sales, Poisson demand, outdating costs, purchase costs, and per unit per period holding costs is considered. Using $(1, T)$ policy in which an order constantly is placed for one unit of product in each constant time interval, the long-run average total cost function including the total outdating, holding, shortage and purchase costs is derived. The objective is to determine the optimal time interval between any two consecutive orders which minimizes the long-run average total cost. It is assumed that the product shelf life is finite and constant, the lead time for an order is constant, and the fixed ordering cost is zero or negligible.

General notations

μ	The demand rate.
π	Cost of a lost sale.
h	Rate of holding cost.
p	Cost of a perished product.
c	Unit price.
τ	Lead time.
m	Product shelf life.
T	Time interval between any two consecutive orders in $(1, T)$ policy.
L	The long-run average number of units in system.
HC	Average holding cost per time unit.
Π	Average shortage cost per time unit.
OC	Average perishing cost per time unit.
PC	Average purchase cost per time unit.
$C(T)$	Average total cost rate, for the $(1, T)$ policy.
$C(S)$	Average total cost rate, for the $(S-1, S)$ policy.
w_{qn}	The queue waiting time of the nth customer.
s_n	The required service time of the nth customer.
W_q	The random variable of the queue waiting time of an arriving customer (product) in steady state.
S	The random variable of the required service time of a customer (product) in steady state.
$\bar{S}$	The random variable of the occurred service time of a customer (product) in steady state.
W_s	A customer (product) average waiting time in the queueing system in steady state.

2.2. Methodology

To obtain the total cost function of the system, one can resort to some concepts of queueing theory. To do this, consider a $D/M/1$ queueing system with impatient customers in which:

(1) The arrival process is the arrival units of product to the system and the inter-arrival times are constant, equal to T,
(2) The customers' (product units') patience time is the product shelf life. That is, the customer gives up whenever his sojourn time is larger than m, and
(3) The inter-demand times, exponentially distributed with mean $1/\mu$, are the required service times of these units.

Hence, the inventory problem can be interpreted as a $D/M/1$ queue with impatient customers, a single channel queueing system in which the arrival process is deterministic with rate $1/T$, the service times have exponential distribution with mean $1/\mu$, and the customer leaves the system whenever his sojourn time is larger than m. It is clear that the number of units in this queueing system is equal to the inventory on hand in the above inventory system.

In the considered queueing system, customers (products) arrive at the instances $0, T, 2T, \ldots, nT, \ldots$ and are served in the order of their arrival. These customers require service for times $s_0, s_1, \ldots, s_n, \ldots$. A customer will wait in the system only for a time not exceeding a fixed time m. We number the customers (arriving units to the inventory system) according to their arrival times, and define w_{qn} as the waiting time in queue of the nth customer (product). The following relation can be obtained between $w_{q(n+1)}$ and w_{qn} using a manner similar to that of Lindley (1952).

$$w_{q(n+1)} = \begin{cases} 0 & \text{if } w_{qn} + s_n \le T \\ w_{qn} + s_n - T & \text{if } T < w_{qn} + s_n < m\,; n = 0, 1, \ldots \\ m - T & \text{if } m \le w_{qn} + s_n \end{cases}$$

or equivalently

$$\begin{aligned} w_{q(n+1)} &= [w_{qn} + \min(s_n, m - w_{qn}) - T]^+ \\ &= [\min(w_{qn} + s_n, m) - T]^+ \quad ; n = 0, 1, \ldots \end{aligned} \tag{1}$$

where $[x]^+ = \max(0, x)$. Thus, the sequence of $\{w_{qn}\}$ forms a Markov chain with the state space $[0, m - T]$.

The probability of a product perishing is $Pr(S + W_q > m)$. Hence, to estimate the perishing costs, we have to obtain the distribution function of W_q.Define $F_n(x) = Pr(W_{qn} \leqslant x)$ as the distribution function of W_{qn}, and let $F(x) = \lim_{n\to\infty} F_n(x)$ when the limit exists. To obtain $F(x)$, we use the analogy of the finite dam model and $D/M/1$ queues with impatient customers.

2.3. The finite dam model

Daley (1964) clarifies the analogy of the queueing system with impatient customers and the finite dam model. Assume a dam with finite capacity m, and consider its storage Z_t for discrete time $t = 0, 1, 2, \ldots$. Suppose in the interval $(t, t + 1)$, an amount of X_t flows into the dam, filling it to the level of $\min(Z_t + X_t, m)$ and any excess water being lost over the spillway. A fixed demand for an amount T occurs just before the instant $t + 1$, and is met as fully as possible by the release of the quantity $\min[T, \min(Z_t + X_t, m)] = \min(T, Z_t + X_t, m)$. Thus an amount Z_{t+1} leaves in the dam, where

$$\begin{aligned} &Z_{t+1} = \min(Z_t + X_t, m) - \min(T, Z_t + X_t, m) \Rightarrow \\ &Z_{t+1} = [\min(Z_t + X_t, m) - T]^+ \end{aligned} \tag{2}$$

If $\{X_t\}$ is assumed to be a sequence of independent non-negative random variables from an exponential distribution with parameter