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In the determination of the most efficient decision making unit in data envelopment analysis ☆

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ABSTRACT

In recent years, several mixed integer linear programming (MILP) models have been proposed for determining the most efficient decision making unit (DMU) in data envelopment analysis. However, most of these models do not determine the most efficient DMU directly; instead, they make use of other less related objectives. This paper introduces a new MILP model that has an objective similar to that of the super-efficiency model. Unlike previous models, the new model's objective is to directly discover the most efficient DMU. Similar to the super-efficiency model, the aim is to choose the most efficient DMU. However, unlike the super-efficiency model, which requires the solution of a linear programming problem for each DMU, the new model requires that only a single MILP problem be solved. Consequently, additional terms in the objective function and more constraints can be easily added to the new model. For example, decision makers can more easily incorporate a secondary objective such as adherence to a publicly stated preference or add assurance region constraints when determining the most efficient DMU. Furthermore, the proposed model is more accurate than two recently proposed models, as shown in two computational examples.

1. Introduction

Data envelopment analysis (DEA) is a method developed by Charnes, Cooper, and Rhodes (1978) to measure the efficiency of decision making units (DMUs). DEA determines the relative efficiency of a group of DMUs that use the same types of input and produce the same types of output. It assigns efficiency scores of less than 1 to inefficient DMUs and scores of strictly 1 to efficient DMUs; thus, all of the efficient DMUs have the same efficiency score. This lack of discriminatory power has motivated numerous researchers to develop different ranking methods for use with DEA (e.g., Andersen & Petersen, 1993; Bal, Orkcu, & Celebioglu, 2008; Doyle & Green, 1994, 1995; Jahanshahloo, Hosseinzadeh Lotfi, Khanmohammadi, Kazemimanesh, & Rezaie, 2010; Jahanshahloo, Vieira Junior, Hosseinzadeh Lotfi, & Akbarian, 2007; Lam, 2010; Lam & Bai, 2011; Sexton, Silkman, & Hogan, 1986; Soltanifar & Lotfi, 2011; Tofallis, 1997; Wu, Liang, & Chen, 2009). Researchers have recently proposed several models for finding the most efficient DMU in DEA (Amin, 2009; Amin & Toloo, 2007; Foroughi, 2011; Toloo & Nalchigar, 2009; Wang & Jiang, 2012).

Wang and Jiang's (2012) model for determining the most efficient DMU features an objective function that maximizes the overall efficiency of all of the DMUs. Whether such a benevolent objective can effectively identify the most efficient DMU remains unclear. Foroughi (2011) proposed a model that maximizes the minimum inefficiencies of all of the DMUs, except the one deemed the most efficient by the model. Again, this aggressive objective may be inappropriate for determining the most efficient DMU. To address these deficiencies, this paper introduces a new model designed to find the most efficient DMU more effectively.

Banker and Chang (2006) pointed out that the super-efficiency model (Andersen & Petersen, 1993) can be used for outlier identification. The original motivation for using the super-efficiency model to identify outliers was the removal of possibly contaminated observations from sample data and the attainment of more reliable efficiency estimates in DEA studies. This procedure for removing outliers assumes that the sample data are contaminated by noise; however, an observation with a high super-efficiency score is not necessarily contaminated. It is possible that the observation itself is truly efficient. For example, consider a case in which all of the DMUs consume the same amount of inputs and produce the same amount of outputs. If one DMU can reduce its consumption of inputs while maintaining the same output level, then it is more efficient than the others. A good model for finding

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the most efficient DMU should be able to identify this model as the most efficient. Thus, using a similar objective to the one used in the super-efficiency model, we formulate the problem of finding the most efficient DMU in DEA as a mixed integer linear programming (MILP) model. In the proposed model, the DMU that achieves the highest possible efficiency ratio is the most efficient DMU. We find that this model outperforms the models proposed by Foroughi (2011) and Wang and Jiang (2012) in identifying the most efficient DMU in computational examples.

This paper provides a good alternative model for finding the most efficient DMU in DEA. It also introduces a simple method that can be used to compare the performance of different models. As different models may choose different DMUs as the most efficient, it is difficult for decision-makers to determine which DMU is actually the most efficient. Furthermore, researchers are often unable to test the performance of their models. In this paper, we provide a simple method for comparing the performance of different models in finding the most efficient DMU. The simplicity of our method means that it can be easily replicated in future studies.

The remainder of the paper is organized as follows. A number of existing models are discussed in Section 2. In Section 3, the newly proposed MILP model is introduced and discussed. The results of computational examples are reported in Section 4, and Section 5 concludes the paper.

2. Existing models for determining the most efficient DMU

Several models for identifying the most efficient DMU have been proposed. Those proposed by Amin and Toloo (2007) and Toloo and Nalchigar (2009) both suffer from the lack of a proper optimization objective (see Foroughi, 2011; Wang & Jiang, 2012), and as a result, any DMU can be identified as the most efficient in both models. Another model, proposed by Amin (2009), suffers from possible infeasibility (see Foroughi, 2011). Wang and Jiang (2012) also pointed out that the solution it provides is not unique and that the model also contains too many variables and nonlinear constraints.

Wang and Jiang (2012) proposed the following model for determining the most efficient DMU under constant returns to scale. Suppose that a group of n DMUs consumes m inputs, x_{ij} ($i = 1, 2,\ldots,m$), and produces s outputs, y_{rj} ($r = 1, 2,\ldots,s$), for $j = 1, 2,\ldots,n$. Model (1) is then stated as follows:

$$\text{Minimize} \sum_{i=1}^{m} v_i \left(\sum_{j=1}^{n} x_{ij}\right) - \sum_{r=1}^{s} u_r \left(\sum_{j=1}^{n} y_{rj}\right) \quad (1.1)$$

$$\text{s.t.} \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - I_j \leqslant 0, \quad j = 1, 2, \ldots, n, \quad (1.2)$$

$$\sum_{j=1}^{n} I_j = 1, \quad (1.3)$$

$$u_r \geqslant \frac{1}{(m+s)\max_j\{y_{rj}\}}, \quad r = 1, 2, \ldots, s, \quad (1.4)$$

$$v_i \geqslant \frac{1}{(m+s)\max_j\{x_{ij}\}}, \quad i = 1, 2, \ldots, m, \quad (1.5)$$

where $I_j \in \{0, 1\}$, $j = 1, 2,\ldots,n$. Constraints (1.4) and (1.5) were proposed by Sueyoshi (1999). Objective function (1.1) maximizes the weighted sum of the outputs and minimizes the weighted sum of the inputs of all of the DMUs. The objective to maximize the overall efficiency of all of the DMUs is actually benevolent in nature (Doyle & Green, 1994), and thus is not directly relevant to the determination of the most efficient DMU.

Model (2), proposed by Foroughi (2011), identifies the most efficient DMU as follows:

$$\text{Maximize } d \quad (2.1)$$

$$\text{s.t.} \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - I_j + d \leqslant 0, \quad j = 1, 2, \ldots, n, \quad (2.2)$$

$$-\sum_{r=1}^{s} u_r y_{rj} + \sum_{i=1}^{m} v_i x_{ij} + I_j \leqslant 1, \quad j = 1, 2, \ldots, n, \quad (2.3)$$

$$\sum_{i=1}^{m} v_i x_{ij} \leqslant 1, \quad j = 1, 2, \ldots, n, \quad (2.4)$$

$$\sum_{j=1}^{n} I_j = 1, \quad (2.5)$$

$$\{v_i\} \in V, \quad (2.6)$$

$$\{u_r\} \in U, \quad (2.7)$$

where $I_j \in \{0, 1\}$, $j = 1, 2,\ldots,n$, and V and U are the sets of all of the acceptable weights. Foroughi (2011) specified the use of $V(\varepsilon)=\{\{v_i\} | v_i \geqslant \varepsilon, \ i = 1,\ldots,m\}$, $U(\varepsilon) = \{\{u_r\} | u_r \geqslant \varepsilon, \ r = 1,\ldots,s\}$, and $\varepsilon \in [0, \varepsilon^*]$, where ε^* is the maximum non-Archimedean (Amin &

Table 1
Inputs and outputs of 19 FLDs.

FLDs	Inputs		Outputs			
Weights	Cost (\$) (v_1)	Adjacency Score (v_2)	Sharp Ratio (u_1)	Flexibility (u_2)	Quality (u_3)	Hand-Carry utility (u_4)
1	20309.56	6405.00	0.4697	0.0113	0.0410	30.89
2	20411.22	5393.00	0.4380	0.0337	0.0484	31.34
3	20280.28	5294.00	0.4392	0.0308	0.0653	30.26
4	20053.20	4450.00	0.3776	0.0245	0.0638	28.03
5	19998.75	4370.00	0.3526	0.0856	0.0484	25.43
6	20193.68	4393.00	0.3674	0.0717	0.0361	29.11
7	19779.76	2862.00	0.2854	0.0245	0.0846	25.29
8	19831.00	5473.00	0.4398	0.0113	0.0125	24.80
9	19608.43	5161.00	0.2868	0.0674	0.0724	24.45
10	20038.10	6078.00	0.6624	0.0856	0.0653	26.45
11	20330.68	4516.00	0.3437	0.0856	0.0638	29.46
12	20155.09	3702.00	0.3526	0.0856	0.0846	28.07
13	19641.86	5762.00	0.2690	0.0337	0.0361	24.58
14	20575.67	4639.00	0.3441	0.0856	0.0638	32.20
15	20687.50	5646.00	0.4326	0.0337	0.0452	33.21
16	20779.75	5507.00	0.3312	0.0856	0.0653	33.60
17	19853.38	3912.00	0.2847	0.0245	0.0638	31.29
18	19853.38	5974.00	0.4398	0.0337	0.0179	25.12
19	20355.00	17402.00	0.4421	0.0856	0.0217	30.02