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Survey

A literature review on the vehicle routing problem with multiple depots

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A B S T R A C T

In this paper, we present a state-of-the-art survey on the vehicle routing problem with multiple depots (MDVRP). Our review considered papers published between 1988 and 2014, in which several variants of the model are studied: time windows, split delivery, heterogeneous fleet, periodic deliveries, and pickup and delivery. The review also classifies the approaches according to the single or multiple objectives that are optimized. Some lines for further research are presented as well.

1. Introduction

Physical distribution is one of the key functions in logistics systems, involving the flow of products from manufacturing plants or distribution centers through the transportation network to consumers. It is a very costly function, especially for the distribution industries. The Operational Research literature has addressed this problem by calling it as the vehicle routing problem (VRP). The VRP is a generic name referring to a class of combinatorial optimization problems in which customers are to be served by a number of vehicles. The vehicles leave the depot, serve customers in the network and return to the depot after completion of their routes. Each customer is described by a certain demand. This problem was firstly proposed in the literature by Dantzig and Ramser (1959). After then, considerable number of variants has been considered: hard, soft and fuzzy service time windows, maximum route length, pickup and delivery, backhauls, etc. (Cordeau, Gendreau, Hertz, Laporte, & Sormany, 2005; Juan, Faulín, Adelanteado, Grasman, & Montoya Torres, 2009; López-Castro & Montoya-Torres, 2011; Montoya-Torres, Alfonso-Lizarazo, Gutiérrez-Franco, & Halabi, 2009; Ozfirat & Ozkarahan, 2010; Thangiah & Salhi, 2001). Solving the VRP is vital in the design of distribution systems in supply chain management.

1.1. VRP versus MDVRP

Formally, the classical vehicle routing problem (VRP) is represented by a directed graph $G(E,V)$, where $V = \{0,1,\ldots,n\}$ represents the set of nodes and E is the set of arcs. The depot is noted to be node $j = 0$, and clients are nodes $j = 1, 2, \ldots, n$, each one with demand $d_j > 0$. Each arc represents a route from node i to node j. The weight of each arc $C_{ij} > 0$ corresponds to the cost (time or even distance) of going from node i to node j. If $C_{ij} = C_{ji}$ then we are facing the symmetric VRP, otherwise the problem is asymmetric. From the complexity point of view, the classical VRP is known to NP-hard since it generalizes the Travelling Salesman Problem (TSP) and the Bin Packing Problem (BPP) which are both well-known NP-hard problems (Garey & Johnson, 1979). A review of mathematical formulations for the classical VRP can be found in the work of Laporte (1992).

In the literature, lots of surveys have been presented analyzing published works on either the classical version (Bodin, 1975; Bodin & Golden, 1981; Desrochers, Lenstra, & Savelsbergh, 1990; Eksioglu, Volkan, & Reisman, 2009; Laporte, 1992; Liong, Wan,

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Khairuddin, & Mourad, 2008; Maffioli, 2002) or its different variants: the capacitated VRP (Baldacci, Toth, & Vigo, 2010; Cordeau, Laporte, Savelsbergh, & Vigo, 2007; Gendreau, Laporte, & Potvin, 2002; Laporte & Nobert, 1987; Laporte & Semet, 2002; Toth & Vigo, 2002), the VRP with heterogeneous fleet of vehicles (Baldacci, Battarra, & Vigo, 2008; Baldacci, Toth, & Vigo, 2007; Baldacci et al., 2010), VRP with time windows (VRPTW), pickup and deliveries and periodic VRP (Solomon & Desrosiers, 1988), dynamic VRP (DVRP) (Psaraftis, 1995), Periodic VRP (PVRP) (Mourgaya & Vanderbeck, 2006), VRP with multiple trips (VPRMT) (Şen & Bülbül, 2008), Split Delivery vehicle routing problem (SDVRP) (Archetti & Speranza, 2008). All of these works consider only one depot. Fig. 1 presents a hierarchy of VRP variants. One of these variants considers a well-known (Crevier, Cordeau, & Laporte, 2007) more realistic situation in which the distributions of goods is done from several depots to final clients. This particular distribution network can be solved as multiple individual single depot VRP's, if and only if clients are evidently clustered around each depot; otherwise a multi-depot-based approach has to be used where clients are to be served from any of the depots using the available fleet of vehicles. In this paper, we consider the variant of the vehicle routing problem known as Multiple Depots Vehicle Routing Problem (MDVRP) in which more than one depot is considered (see Fig. 2). The reader must note that most exact algorithms for solving the classical VRP model are difficult to be adapted for solving the MDVRP.

According to Renaud, Laporte, and Boctor (1996), the MDVRP can be formally described as follows. Let $G = (V,E)$ be a graph, where V is the set of nodes and E is the set of arcs or edges connecting each pair of nodes. The set V is further partitioned into two subsets: $V_c = \{v_1, v_2, \ldots, v_N\}$ which is the set of customers to be served; and $V_d = \{v_{N+1}, v_{N+2}, \ldots, v_M\}$ which is the set of depots. Each customer $v_i \in V_c$ has a nonnegative demand d_i. Each arc belong to the set E has associated a cost, distance or travel time c_{ij}. There are a total of K vehicles, each one with capacity P_k. The problem consists on determining a set of vehicle routes in such a way that: (i) each vehicle route starts and ends at the same depot, (ii) each customer is serviced exactly once by a vehicle, (iii) the total demand of each route does not exceed the vehicle capacity, and (iv) the total cost of the distribution is minimized. According to Kulkarni and Bhave (1985), a mathematical model of the MDVRP requires the definition of binary decision variables x_{ijk} to be equal to 1 if the pair of nodes i and j are in the route of vehicle k, and 0 otherwise. Auxiliary variables y_i are required in order to avoid subtour elimination. According to this last reference, the model is as follows:

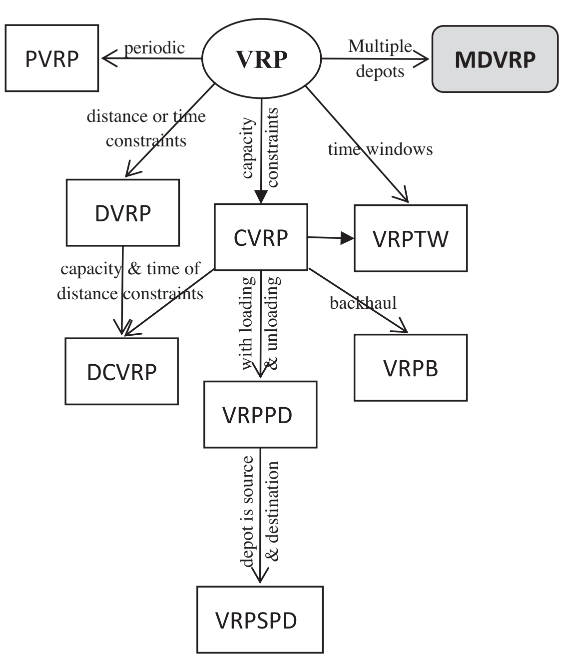

Fig. 1. Different variants of the VRP (adapted from Weise et al., 2010).

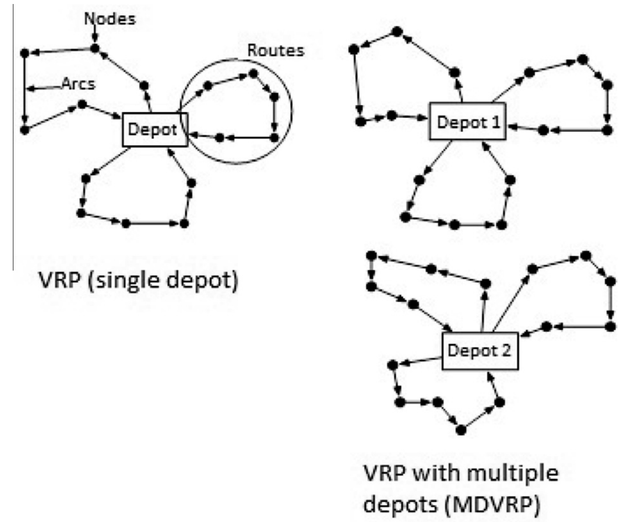

Fig. 2. Comparison between VRP versus MDVRP.

$$\text{Minimize} \sum_{i=1}^{N+M} \sum_{j=1}^{N+M} \sum_{k=1}^{K} (c_{ij} x_{ijk}) \tag{1}$$

Subject to :

$$\sum_{i=1}^{N+M} \sum_{k=1}^{K} x_{ijk} = 1 \quad j = 1, \ldots, N \tag{2}$$

$$\sum_{j=1}^{N+M} \sum_{k=1}^{K} x_{ijk} = 1 \quad i = 1, \ldots, N \tag{3}$$

$$\sum_{i=1}^{N+M} x_{ihk} - \sum_{j=1}^{N+M} x_{hjk} = 0 \quad \begin{matrix} k = 1, \ldots, K \\ h = 1, \ldots, N+M \end{matrix} \tag{4}$$

$$\sum_{i=1}^{N+M} Q_i \sum_{j=1}^{N+M} x_{ijk} \leqslant P_k \quad k = 1, \ldots, K \tag{5}$$

$$\sum_{i=1}^{N+M} \sum_{j=1}^{N+M} c_{ij} x_{ijk} \leqslant T_k \quad k = 1, \ldots, K \tag{6}$$

$$\sum_{i=N+1}^{N+M} \sum_{j=1}^{N} x_{ijk} \leqslant 1 \quad k = 1, \ldots, K \tag{7}$$

$$\sum_{j=N+1}^{N+M} \sum_{i=1}^{N} x_{ijk} \leqslant 1 \quad k = 1, \ldots, K \tag{8}$$

$$y_i - y_j + (M+N) x_{ijk} \leqslant N + M - 1 \quad \text{For } 1 \leqslant i \neq j \leqslant N \text{ and } 1 \leqslant k \leqslant K \tag{9}$$

$$x_{ijk} \in \{0,1\} \quad \forall i,j,k \tag{10}$$

In this formulation, Constraints (2) and (3) ensure that each customer is served by one and only one vehicle. Route continuity is represented by Constraints (4). The sets of constraints (5) and (6) are the vehicle capacity and total route cost constraints. Vehicle availability is verified by Constraints (7) and (8) and subtour elimination is provided by Constraints (9). In this formulation, it is assumed that total demand at each node is either less than or at