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Single-machine scheduling with a variable maintenance activity

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ABSTRACT

We consider the problem of scheduling a maintenance activity and jobs on a single machine, where the maintenance activity must start before a given deadline and the maintenance duration increases with its starting time. We provide polynomial-time algorithms to solve the problems to minimize the makespan, sum of completion times, maximum lateness, and number of tardy jobs.

1. Introduction

Scheduling jobs with machine maintenance in a manufacturing system has received considerable attention in the recent past. During a maintenance period, job processing is stopped while maintenance operations such as cleaning, recharging, refilling, or partial replacement of tools are performed. Usually, we use two parameters to define a maintenance activity. One is its starting time and the other is its duration. According to the two parameters, scheduling with machine maintenance falls roughly into two categories, i.e., scheduling with fixed maintenance and scheduling with variable maintenance. In scheduling with fixed maintenance, the starting time and the duration of maintenance are both determined in advance. Since the 1990s, various scheduling models with fixed maintenance in different machine environments and job characteristics have been investigated extensively in the literature. For the related important papers, see e.g., (Cheng, Hsu, & Yang, 2011; Epstein et al., 2012; Fu, Huo, & Zhao, 2009; Ji, He, & Cheng, 2007; Ji, Ge, Chen, & Cheng, 2013; Kacem, Chu, & Souissi, 2008; Kacem, 2008; Kacem & Chu, 2008; Kellerer, Kubzin, & Strusevich, 2009; Lee & Liman, 1992; Lee, 1996, 1997, 1999; Lee & Chen, 2000; Mati, 2010; Mor & Mosheiov, 2012; Moncel, Thiery, & Waserhole, 2014; Qi, Chen, & Tu, 1999 and Wang, Sun, & Chu, 2005). For the related surveys, we refer the reader to Ma, Chu, and Zuo (2010) and Schmidt (2000). For scheduling with variable maintenance, the starting time is a decision variable, which is determined by the scheduler, and the maintenance duration is a positive and nondecreasing function of its starting time. To the best of our knowledge, Kubzin and Strusevich (2006) first considered scheduling problems with variable maintenance in the two-machine shop setting. Studying makespan minimization in a two-machine flow shop and a two-machine open shop, they showed that the open shop problem is polynomially solvable for quite general functions defining the maintenance duration while the flow shop problem is binary NP-hard and pseudo-polynomially solvable by dynamic programming. Furthermore, they presented a fully polynomial-time approximation scheme and a fast 3/2-approximation algorithm for this problem. Mosheiov and Sidney (2010) studied a problem of scheduling an optional maintenance activity on a single machine, where the maintenance duration is assumed to be a nondecreasing linear function of its starting time and the processing time of a job scheduled after the maintenance activity becomes smaller than it is scheduled before the maintenance activity. They showed that the problems to minimize the makespan, flowtime, maximum lateness, total earliness, tardiness and due-date costs, and number of tardy jobs are all polynomially solvable. Xu, Yin, and Li (2010) considered two scheduling problems with machine maintenance under the assumption that the maintenance duration is an increasing linear function of the total processing time of the jobs that are processed after the machine's last maintenance activity. The first problem concerns parallel-machine scheduling to minimize the completion time of

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the last finished maintenance, where the length of the time interval between any two consecutive maintenance activities is between two given positive numbers. The second problem deals with single-machine scheduling to minimize the completion time of the last finished job, where the length of the time interval between any two consecutive maintenance activities is fixed. They proposed two approximation algorithms for the considered problems and analyze their performance. Luo, Chen, and Zhang (2010) considered the problem of scheduling weighted jobs on a single machine with a maintenance activity whose starting time must be prior to a given deadline and whose duration is a nondecreasing function of its starting time. Studying the problem to minimize the total weighted job completion time, they showed that it is weakly NP-hard. For the case where the maintenance duration is a concave (and nondecreasing) function of its starting time, they provided two approximation algorithms with an approximation ratio of 2 and at most $1 + \sqrt{2}/2 + \epsilon$, respectively.

In this paper we study some problems of scheduling a variable maintenance activity and jobs on a single machine. Studying the problems to minimize the makespan, sum of completion times, maximum lateness, and number of tardy jobs, we show that they are all polynomially solvable.

As a practical example for the proposed problems, we consider steel strip production in a steel plant (Tang, Ren, & Yang, 2014), in which steel slabs must pass a re-heat furnace before they are rolled into strips. We can regard the slabs as "jobs" and the re-heat furnace as the "machine". To ensure that the re-heat furnace functions normally, we must clean it and refill its fuels prior to a given deadline. However, the duration of cleaning and refilling the furnace depends on the total processing time of the steel slabs that have been handled, which can be regarded as "a variable maintenance activity".

The remainder of the paper is organized as follows: In the next section we introduce and formulate the problems. In Sections 3–6, we solve the problems to minimize the makespan, sum of completion times, maximum lateness, and number of tardy jobs, respectively. We conclude the paper in Section 7.

2. Problem statement

The problem can be described as follows: There is a set of n independent jobs $\mathcal{J} = \{1, 2, \ldots, n\}$ to be non-preemptively processed on a single machine, all of which are available for processing at time zero. The machine must undergo mandatory maintenance activity prior to a given deadline and the maintenance duration increases with its starting time, i.e., the later the maintenance activity begins, the longer is its duration.

Let p_j denote the job processing time of job j, $j = 1, 2, \ldots, n$, s denote the starting time of maintenance, and s_d denote the deadline for the starting time of maintenance. Clearly, we have $s \leqslant s_d$. The maintenance duration l is a positive and nondecreasing function of the starting time of maintenance s, i.e., $l = f(s)$, where $f(s)$ is positive and nondecreasing function of s.

For a given schedule, let C_j denote the completion time of job j, $j = 1, 2, \ldots, n$. Then its lateness is $L_j = C_j - d_j$, where d_j denotes the due date of job j, $j = 1, 2, \ldots, n$, and the maximum lateness is L_{max} $(= \max_{j=1,2\ldots,n} L_j)$. Furthermore, let $U_j = 1$ if $C_j > d_j$ (a tardy job), and $U_j = 0$ if $C_j \leqslant d_j$ (an on-time job), $j = 1, 2, \ldots, n$.

The task is to determine the starting time of the maintenance activity and sequence all the jobs on the machine such that the makespan, sum of completion times, maximum lateness, or number of tardy jobs is minimized.

Formally, we use "VM" to denote a variable maintenance activity. Using the three-field classification scheme for scheduling problems proposed by Graham, Lawler, Lenstra, and Rinnooy Kan (1979), we denote the four problems under study as $1, VM||C_{max}$, $1, VM||\sum_j C_j$, $1, VM||L_{max}$, and $1, VM||\sum_j U_j$.

To facilitate the understanding of the reader, we summarize the notations used in the entire paper as follows:

n	Number of jobs;
$\mathcal{J}$	The set of all the jobs, $\mathcal{J} = \{1, 2, \ldots, n\}$;
p_j	The processing time of job j;
d_j	The due date of job j;
C_j	The completion time of job j;
L_j	The lateness of job j;
U_j	The tardiness indicator of job j;
s	The starting time of maintenance activity;
s_d	The deadline of maintenance activity;
l	The duration of maintenance activity ($l = f(s)$, where $f(\cdot)$ is a positive and nondecreasing function on s);
C_{max}	The maximum completion time (makespan); and
L_{max}	The maximum lateness.

Without loss of generality, we assume $s_d < \sum_{j=1}^n p_j$ throughout the paper. Otherwise, all the above problems are equivalent to the corresponding counterparts without maintenance.

The following lemma provides an obvious property for an optimal schedule for all the problems under study.

Lemma 1. *There exists an optimal schedule without idle time among the jobs and the maintenance activity.*

Proof. If there exists idle time between jobs, we can move the jobs after the idle time earlier to clear the gap without increasing the objective value.

If the maintenance activity is scheduled δ times after some job, we can move the maintenance activity δ times forward to clear the idle time. Then the maintenance duration is shortened and the completion times of the jobs after the maintenance activity all become smaller.

If the maintenance activity is scheduled δ times just before some job, we can move the job δ times earlier without increasing the objective value.

The above analysis shows that we can always clear the idle time, if any, among the jobs and the maintenance activity without increasing the objective value, so Lemma 1 holds. $\square$

3. Minimizing the makespan

For this problem, notice that $s_d < \sum_{j=1}^n p_j$. Clearly starting the maintenance activity at time zero (i.e., the maintenance duration is minimized) and scheduling all the jobs in an arbitrary order after the maintenance activity yield an optimal schedule. Hence, starting maintenance at time zero solves problem $1, VM||C_{max}$ in $O(n)$ time.

4. Minimizing the sum of completion times

Recall that the shortest processing time (SPT) rule solves single-machine scheduling to minimize the sum of completion times without maintenance. To solve problem $1, VM||\sum_j C_j$, we derive the optimal job order as follows:

Lemma 2. *For problem $1, VM||\sum_j C_j$, there exists an optimal schedule in which the jobs are scheduled in the SPT order.*