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An evolutionary approach for multi-objective vehicle routing problems with backhauls ☆

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ABSTRACT

The vehicle routing problem (VRP) is an important aspect of transportation logistics with many variants. This paper studies the VRP with backhauls (VRPB) in which the set of customers is partitioned into two subsets: linehaul customers requiring a quantity of product to be delivered, and backhaul customers with a quantity to be picked up. The basic VRPB involves finding a collection of routes with minimum cost, such that all linehaul and backhaul customers are serviced. A common variant is the VRP with selective backhauls (VRPSB), where the collection from backhaul customers is optional. For most real world applications, the number of vehicles, the total travel cost, and the uncollected backhauls are all important objectives to be minimized, so the VRPB needs to be tackled as a multi-objective problem. In this paper, a similarity-based selection evolutionary algorithm approach is proposed for finding improved multi-objective solutions for VRPB, VRPSB, and two further generalizations of them, with fully multi-objective performance evaluation.

1. Introduction

The main objective of the vehicle routing problem (VRP) is to obtain the lowest-cost set of routes to deliver demand to customers from a depot, and sometimes to also collect a quantity of product from customers. Since Dantzig and Ramser (1959) introduced the VRP more than 50 years ago, it has been the subject of extensive research and has become one of the most studied combinatorial optimization problems. As observed by Golden et al. (2008, Preface),

> "vehicle routing may be the single biggest success story in operations research. For example, each day 103,500 drivers at UPS follow computer-generated routes. The drivers visit 7.9 million customers and handle an average of 15.6 million packages",

so it is of tremendous practical importance for transportation logistics. In fact, because of the diversity of operating rules and constraints encountered in real-world applications, numerous variants of the problem exist, and the VRP should really be viewed as a whole class of problems (Laporte, 2009).

One particularly common variant is the VRP *with backhauls* (VRPB), which involves both delivery and collection points (Toth & Vigo, 2001). *Linehaul* customers are sites with a demand of goods, and deliveries have to be made to them from the depot or distribution center. *Backhaul* customers are points from which a quantity of goods has to be collected and taken to the depot. A practical example of this is the manufacturing industry, where factories are the linehaul customers, and raw materials and components are supplied by the backhaul customers.

The general problem consists of designing a set of routes with minimum cost to service the given linehaul and backhaul customers. Since the VRP was originally proposed as a generalization of the traveling salesman problem (Dantzig & Ramser, 1959), the cost has primarily been associated with the number of routes (or vehicles) and the total travel distance (or time), but there are several other potential sources of cost (Jozefowiez, Semet, & Talbi, 2008). In practice, given the constraints, minimization of the travel cost often results in an increased number of routes, so if both objectives are considered to be of importance, the VRPB really needs to be tackled as a bi-objective problem. Moreover, if a revenue is associated with each backhaul customer, and these are considered optional, that is another objective which needs to be taken into

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account, and the VRPB should be tackled as a tri-objective problem. This last variant is known as VRP with *selective backhauls* (VRPSB) (Baldacci, Bartolini, & Laporte, 2010).

Exact methods have been devised to find optimal solutions for relatively small instances of the VRPB (Toth & Vigo, 1997; Mingozzi, Giorgi, & Baldacci, 1999), but, since it belongs to the NP-hard class of problems (Lenstra & Kan, 1981), the computation time required increases considerably for larger instances. For realistically sized problems, one is therefore forced to use heuristic approaches. There have been many past studies which have solved the VRPB as a single-objective problem using heuristic and metaheuristic methods, such as tabu search (Glover, 1989; Glover, 1990) and ant colony algorithms (Dorigo, Maniezzo, & Colorni, 1996). However, very few studies have considered the VRPB and VRPSB as multi-objective problems. One particularly effective approach, that has not been fully investigated before, involves using evolutionary algorithms (EAs) which can generate a whole population of solutions to cover the full range of trade-offs among objectives.

In a preliminary study, a simple evolutionary algorithm for solving standard benchmark instances of the VRPB and VRPSB variants was introduced with promising results (Garcia-Najera, 2012). This paper now presents an improved evolutionary approach, involving the optimization of two and three objectives for well-known instances of the VRPB and two further generalizations of the problem. It builds on an earlier application of evolutionary computation techniques to the VRP *with time windows* (Garcia-Najera & Bullinaria, 2011), that introduced a novel selection process involving solution dissimilarity to generate solution sets with better coverage of the full range of trade-off possibilities. However, application to the VRPB is not straightforward, because it requires the formulation of supplementary problem-specific evolutionary operators, and a careful multi-objective evaluation of the solutions generated. Comparisons with existing single-objective algorithms are first provided, and then fully multi-objective performance metrics are used to explore the properties of the current benchmark instances, and demonstrate the advantages of the VRPB-specific similarity-based selection processes over the general purpose crowding mechanism of the widely used NSGA-II (Deb, Pratap, Agarwal, & Meyarivan, 2002) and over the decomposition approach of the successful MOEA/D (Zhang & Li, 2007). Moreover, the multi-objective performance is further analyzed by studying the performance of the algorithm when different objectives are considered for optimization.

The remainder of this paper is organized as follows: The next section describes formally the main VRPB variants, and Section 3 surveys the principal previous studies of them. Section 4 reviews the key concepts of multi-objective optimization, and describes the multi-objective performance metric used later. The proposed approach for solving the VRPB as a multi-objective problem, and its extension for solving VRPSB, are described in Section 5. Then, Section 6 presents results from the proposed algorithm for a range of benchmark problem instances, and provides comparisons with previously published algorithms. Finally, some conclusions are provided in Section 7.

2. VRP with backhauls

The basic version of the VRP is the *capacitated* VRP (CVRP), which considers a set $\mathcal{V} = \{0, \ldots, N\}$ of $N + 1$ vertices, where the subset $\mathcal{V}' = \mathcal{V} \setminus \{0\} = \{1, \ldots, N\}$ are the customers. Each customer $i \in \mathcal{V}'$ is geographically located at coordinates (x_i, y_i) and has a demand of goods $d_i > 0$ to be delivered. The special vertex 0, located at (x_0, y_0), with $d_0 = 0$, is the *depot* from which the customers are serviced. There is a homogeneous fleet of K vehicles available to deliver demand to customers, departing from and arriving at the depot, and having capacity $Q \geqslant \max\{d_i : i = 1, \ldots, N\}$. The travel from vertex i to vertex j has an associated cost c_{ij}, and the core problem consists of finding a set of routes which minimizes the total travel cost.

The VRP *with backhauls* (VRPB) is an extension of the CVRP, where the customers are grouped into *linehaul* customers, which have a demand of goods, and *backhaul* customers, from which a quantity of goods has to be collected. Thus, an instance of the VRPB can be formally defined as a set $\mathcal{V} = \{0, \ldots, N_L, N_L + 1, \ldots, N_L + N_B\}$ of $N + 1$ vertices, representing the depot and $N = N_L + N_B$ customers (Toth & Vigo, 1997). The customers are represented by the vertices in subset $\mathcal{V}' = \mathcal{V} \setminus \{0\} = \{1, \ldots, N_L, N_L + 1, \ldots, N_L + N_B\}$, and each customer $i \in \mathcal{V}'$ is geographically located at coordinates (x_i, y_i). The subset $\mathcal{V}_L = \{1, \ldots, N_L\}$ corresponds to linehaul customers, where each customer $i \in \mathcal{V}_L$ has a demand of goods $d_i > 0$ to be delivered. The subset $\mathcal{V}_B = \{N_L + 1, \ldots, N_L + N_B\}$ represents the backhaul customers, where each customer $i \in \mathcal{V}_B$ has a supply $s_i > 0$ to be collected. A homogeneous fleet of K vehicles is available to deliver and collect goods to and from customers, departing from and arriving at the depot, and having capacity $Q \geqslant \max\{\max\{d_i : i \in \mathcal{V}_L\}, \max\{s_i : i \in \mathcal{V}_B\}\}$.

The main objective is to find a set of K routes which minimize the total travel cost, subject to the following conditions (Duhamel, Potvin, & Rousseau, 1997):

(i) each vehicle services exactly one route,
(ii) each customer is visited exactly once by one vehicle,
(iii) a route is not allowed to consist entirely of backhaul customers,
(iv) backhaul customers in a route can only be served after all linehaul customers, and
(v) for each route, the total load associated with linehaul or backhaul customers cannot exceed the vehicle capacity Q.

The fourth constraint corresponds to the fact that most vehicles are rear-loaded and rearrangement of vehicle loads at delivery points is generally deemed infeasible (Goetschalckx & Jacobs-Blecha, 1989), and also accommodates the fact that linehaul customers frequently prefer early deliveries, while backhaul customers prefer late collections (Ropke & Pisinger, 2006).

Some interesting practical generalizations of the VRPB involve relaxing the third and fourth constraints. One of them is known as the VRP *with mixed backhauls* (VRPMB), which allows backhaul customers to be serviced at any point within a route. That is, linehaul and backhaul customers can be mixed freely within a route, and routes can consist only of backhaul customers. Another variation is the VRP *with simultaneous deliveries and pickups* (VRPSDP), in which customers simultaneously demand goods from and supply goods to the depot. In this case, both delivery and pickup should occur at customers, and they should be performed simultaneously so that each customer is only visited once by a vehicle, and unloading is obviously done before loading.

There are further generalizations of these problems, where all the linehaul customers must be visited, but picking up from backhaul customers is optional. These are the VRP *with selective backhauls* (VRPSB) (Baldacci et al., 2010), and the VRP *with mixed and selective backhauls* (VRPMSB). In these problems, each backhaul customer $i \in \mathcal{V}_B$ has an associated profit $p_i > 0$, and consequently

$$P = \sum_{i \in \mathcal{V}_B} p_i \tag{1}$$

is the total possible profit. The VRPSB and VRPMSB can simply involve determining a set of vehicle routes with minimum net cost (i.e., routing cost minus collected profit), given that visiting