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Multi-attribute group decision making method based on geometric Bonferroni mean operator of trapezoidal interval type-2 fuzzy numbers ☆

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ABSTRACT

In this paper, we investigate the fuzzy multi-attribute group decision making (FMAGDM) problems in which all the information provided by the decision makers (DMs) is expressed as the trapezoidal interval type-2 fuzzy sets (IT2 FS). We introduce the concepts of interval possibility mean value and present a new method for calculating the possibility degree of two trapezoidal IT2 FS. Then, we develop two aggregation techniques called the trapezoidal interval type-2 fuzzy geometric Bonferroni mean (TIT2FGBM) operator and the trapezoidal interval type-2 fuzzy weighted geometric Bonferroni mean (TIT2FWGBM) operator. We study its properties and discuss its special cases. Based on the TIT2FWGBM operator and the possibility degree, the method of FMAGDM with trapezoidal interval type-2 fuzzy information is proposed. Finally, an illustrative example is given to verify the developed approaches and to demonstrate their practicality and effectiveness.

1. Introduction

Fuzzy multi-attributes group decision making (FMAGDM) problem is to find the most desirable alternative from a set of feasible alternatives, where the information provided by a group of decision makers is usually uncertain or fuzzy due to the increasing complexity of the socio-economic environment and the vagueness of inherent subjective nature of human thinking. In recent years, some methods have been presented to deal with FMAGDM problems based on traditional type-1 fuzzy sets (T1 FS). Chen (2000) presented an extension of the TOPSIS method for FMAGDM problem. Chen (2001) presented a method to evaluate the rate of aggregative risk in software development using fuzzy sets under the fuzzy group decision making environment. Li (2007) presented a method for FMADM based on the particular measure of closeness to ideal solution which is developed from the fuzzy weighted Minkowski distance used as an aggregating function in a compromise programming method. Xu (2007) established a practical interactive procedure for solving the FMADM problems, in which the information about attribute weights is partly known. Fan and Liu (2010) presented a method for group decision-making based on the multi-granularity uncertain linguistic information. Lin and Wu (2008) presented a causal analytical method for group decision-making in the fuzzy environment. Tsai and Wang (2008) presented a method for computing coordination based fuzzy group decision-making (CC-FGDM) for web service. Wang and Lin (2003) presented a method for FMAGDM to select configuration items for software development. Wu and Chen (2007) presented a method for maximizing deviation for group multi-attributes decision-making in a linguistic environment. Li (2010) presented some different distances measure and develop a method for solving FMAGDM problems with non-homogeneous information. However, the above FMAGDM methods are based on traditional T1 FS.

The concept of type-2 fuzzy sets (T2 FS), initially introduced by Zadeh (1975), can be regarded as an extension of the concept of T1 FS. The main difference between the two kinds of fuzzy sets is that the memberships of T1 FS are crisp numbers whereas the memberships of T2 FS are T1 FS (Wu & Mendel, 2007); hence, T2 FS involve more uncertainties than T1 FS. Since its introduction, type-2 fuzzy sets are receiving more and more attention. Because the computational complexity of using general T2 FS is very high, to date, interval type-2 fuzzy sets (IT2 FS) (Mendel, John, & Liu, 2006) are the most widely used type-2 fuzzy sets and have been successfully applied to many practical fields (Mendel, 2001; Mendel & Wu, 2006, 2007; Wu & Mendel, 2008a,b). In particular, some authors have applied IT2 FS theory to the field of FMAGDM. Wu and Mendel (2008a,b, 2009) presented a method using the linguistic weighted average and IT2 FS for handling fuzzy multiple criteria hierarchical group decision-making problems. Chen and Lee (2010a, 2010b) presented a method for FMAGDM based on ranking

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values and the arithmetic operations of IT2 FS. Chen and Lee (2010a, 2010b) presented an interval type-2 fuzzy TOPSIS method to handle FMAGDM problems based on IT2 FS. Wang and Liu (2012) investigated the FMAGDM problems under IT2 fuzzy environment, and developed an approach to handling the situations where the attribute values are characterized by IT2 FS, and the information about attribute weights is partially known. Chen and Yang (2012) proposed a method for FMAGDM based on the ranking method of IT2 FS. Zhang and Zhang (2013) proposed a novel approach to FMAGDM by using trapezoidal interval type-2 fuzzy soft sets. Chen and Chang (2013) developed an extended QUALIFLEX method for handling MAGDM problems in the context of IT2 FS and applications to medical decision making. Hu, Zhang, Chen, and Liu (2013) proposed a method based on possibility degree to solve multi-criteria decision making problems in which the criteria value takes the form of interval type-2 fuzzy number.

In this paper, the traditional Bonferroni mean (Bonferroni, 1950) operator has been extended to the interval type-2 fuzzy environment to organize and model the uncertainties better within multi-attribute decision analysis. We present a new method to deal with fuzzy multi-attribute group decision-making problems based on the trapezoidal interval type-2 fuzzy weighted geometric Bonferroni mean operator and the possibility degree of IT2 FS. The remainder of this paper is organized as follows. In Section 2, we give a review of basic concepts and operations related to IT2 FS. In Section 3, we introduce the concepts of lower and upper possibility mean value of IT2 FS. Then, we present a new method for calculating the possibility degree of two IT2 FS based on the interval-valued possibility mean values. In Section 4, the trapezoidal interval type-2 fuzzy geometric Bonferroni mean (TIT2FGBM) operator and the trapezoidal interval type-2 fuzzy weighted geometric Bonferroni mean (TIT2FWGBM) operator are developed, some desirable properties of these operators are studied and some special cases are discussed. Section 5 introduces a procedure for FMAGDM problem based on TIT2FWGBM operator and the possibility degree of two IT2 FS. Section 6 we use global supplier selection problem to illustrate the proposed method. The conclusions are discussed in Section 7.

2. The basic concepts and arithmetic operations of IT2 FS

In this section, the basic concepts and arithmetic operations of IT2 FS are introduced below to facilitate future discussions.

Definition 1 (Mendel & Wu, 2006). A T2 FS $\tilde{A}$ in the universe of discourse X can be represented by a type-2 membership function $\mu_{\tilde{A}}$, shown as follows:

$$\tilde{A} = \{((x,u),\mu_{\tilde{A}}(x,u)) | \forall x \in X, \forall u \in J_X \subseteq [0,1], 0 \leqslant \mu_{\tilde{A}}(x,u) \leqslant 1\} \tag{1}$$

where $J_X \subseteq [0,1]$. The T2 FS $\tilde{A}$ also can be represented as follows:

$$\tilde{A} = \int_{x\in X} \int_{u \in J_X} \mu_{\tilde{A}}(x,u)/(x,u) \tag{2}$$

where $J_X \subseteq [0,1]$ and $\iint$ denotes the union over all admissible x and u.

Definition 2 (Mendel & Wu, 2006). Let $\tilde{A}$ be a T2 FS in the universe of discourse X represented by the type-2 membership function $\mu_{\tilde{A}}$. If all $\mu_{\tilde{A}}(x,u) = 1$, then $\tilde{A}$ is called an IT2 FS. An IT2 FS $\tilde{A}$ can be regarded as a special case of a T2 FS, shown as follows:

$$\tilde{A} = \int_{x\in X} \int_{u \in J_X} 1/(x,u) \tag{3}$$

where $J_X \subseteq [0,1]$.

Definition 3 (Mendel & Wu, 2006). The upper membership function and the lower membership function of an IT2 FS are type-1 membership functions, respectively.

In this paper, we present a method to use IT2 FS for handling FMAGDM problems, where the reference points and the heights of the upper and the lower membership functions of IT2 FS are used to characterize IT2 FS. A trapezoidal IT2 FS $\tilde{A}_i$ is shown in Fig. 1.

Definition 4 (Lee & Chen, 2008). The addition operation between the trapezoidal IT2 FS $\tilde{A}_1 = (\tilde{A}_1^U, \tilde{A}_1^L) = ((a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; h_1^U), (a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; h_1^L))$ and $\tilde{A}_2 = (\tilde{A}_2^U, \tilde{A}_2^L) = ((a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; h_2^U), (a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; h_2^L))$ is defined as follows:

$$\begin{aligned}\tilde{A}_1 \oplus \tilde{A}_2 &= (\tilde{A}_1^U, \tilde{A}_1^L) \oplus (\tilde{A}_2^U, \tilde{A}_2^L) \\ &= ((a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U \\ &\quad + a_{24}^U; \min\{h_1^U, h_2^U\}), (a_{11}^L + a_{21}^L, a_{12}^L + a_{22}^L, a_{13}^L \\ &\quad + a_{23}^L, a_{14}^L + a_{24}^L; \min\{h_1^L, h_2^L\}))\end{aligned} \tag{4}$$

Definition 5 (Lee & Chen, 2008). The multiplication operation between the trapezoidal IT2 FS $\tilde{A}_1 = (\tilde{A}_1^U, \tilde{A}_1^L) = ((a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; h_1^U), (a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; h_1^L))$ and $\tilde{A}_2 = (\tilde{A}_2^U, \tilde{A}_2^L) = ((a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; h_2^U), (a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; h_2^L))$ is defined as follows:

$$\begin{aligned}\tilde{A}_1 \otimes \tilde{A}_2 &= (\tilde{A}_1^U, \tilde{A}_1^L) \otimes (\tilde{A}_2^U, \tilde{A}_2^L) \\ &= ((a_{11}^U \times a_{21}^U, a_{12}^U \times a_{22}^U, a_{13}^U \times a_{23}^U, a_{14}^U \\ &\quad \times a_{24}^U; \min\{h_1^U, h_2^U\}), (a_{11}^L \times a_{21}^L, a_{12}^L \times a_{22}^L, a_{13}^L \\ &\quad \times a_{23}^L, a_{14}^L \times a_{24}^L; \min\{h_1^L, h_2^L\}))\end{aligned} \tag{5}$$

Definition 6 (Lee & Chen, 2008). The arithmetic operation between the trapezoidal IT2 FS $\tilde{A}_1 = (\tilde{A}_1^U, \tilde{A}_1^L) = ((a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; h_1^U), (a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; h_1^L))$ and the crisp value k is defined as follows:

$$\begin{aligned}k\tilde{A}_1 &= k(\tilde{A}_1^U, \tilde{A}_1^L) \\ &= \left(ka_{11}^U, ka_{12}^U, ka_{13}^U, ka_{14}^U; h_1^U\right), \left(ka_{11}^L, ka_{12}^L, ka_{13}^L, ka_{14}^L; h_1^L\right)\end{aligned} \tag{6}$$

where $k>0$.

Definition 7. The exponent operation of the trapezoidal IT2 FS $\tilde{A}_1 = (\tilde{A}_1^U, \tilde{A}_1^L) = ((a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; h_1^U), (a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; h_1^L))$ is defined as follows:

$$\begin{aligned}(\tilde{A}_1)^k = (\tilde{A}_1^U, \tilde{A}_1^L)^k &= \left((a_{11}^U)^k, (a_{12}^U)^k, (a_{13}^U)^k, (a_{14}^U)^k; h_1^U\right), \\ &\quad \left((a_{11}^L)^k, (a_{11}^L)^k, (a_{12}^L)^k, (a_{13}^L)^k, (a_{14}^L)^k; h_1^L\right)\end{aligned} \tag{7}$$

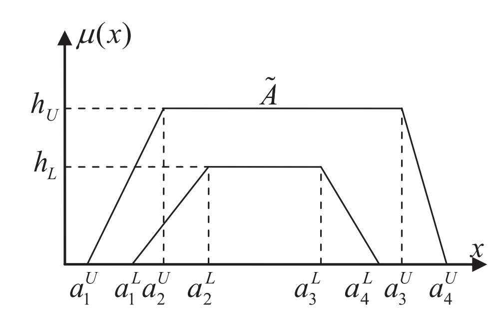

Fig. 1. The upper trapezoidal membership function $\tilde{A}_U$ and the lower trapezoidal membership function $\tilde{A}_L$ of the IT2 FS $\tilde{A}$.