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A coarse-to-fine quasi-physical optimization method for solving the circle packing problem with equilibrium constraints

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A B S T R A C T

This paper addresses an important extension of the circle packing problem (CPP), the circle packing problem with equilibrium constraints (CPPEC). It considers the dense packing of n circular disks in a large circular container at the same time satisfying the equilibrium constraints. Under the industrial background of the layout design on satellite modules, this NP-hard global optimization problem is important in both theory and practice. We introduce two new quasi-physical models for solving CPPEC in this paper. One is to mimic the elastic movement driven by repelling forces from extruded disks, the other is to simulate a whole translation movement of the disks driven by a pulling force from an imaginative elastic rope connecting the centroid of the disks and the center of the container. Then, inspired by the coarse-to-fine control strategy in the manufacture industry, we propose a coarse-to-fine quasi-physical (CFQP) optimization method that adopts the two quasi-physical models for the quasi-physical descent procedure and combines a basin hopping with tabu method for the search procedure. In this way, not only could CFQP take into account the diversity of the search space to facilitate the global search, but it also does fine search to find the corresponding local minimum in a promising local area. Experiments were on two sets of 11 representative test instances. Computational results showed that CFQP achieved new and better results on four instances, at the same time it matched the current best records on the other six (accurate to 0.0001). Moreover, CFQP resulted in smaller equilibrium deviations than that of others published in the literature. In addition, we generated 34 new CPPEC instances basing on the CPP benchmarks, and provided computational results on the two sets of 34 new CPPEC instances, and the container radii obtained are close to the published results on CPP.

1. Introduction

With diverse applications in VLSI design, transportation, aerospace and other industries, many packing problems are concerned with dense packing of n items in a larger container without overlapping. Especially, the circle packing problem (CPP), whose items are equal or unequal circles and container is in the shape of circle or rectangle, is the most typical packing problem. Owing to the exponential scale of the solution space, it is unrealistic to solve these NP-hard problems by solely relying on mathematics, and researchers have tried to get inspiration from the biological world, the physical world and the human society to propose heuristic yet efficient algorithms.

The circle packing problem (CPP) can be classified into two categories that have been extensively investigated and many efficient approaches exist. (1) Packing equal or unequal circles into a larger circle. (2) Packing equal or unequal circles into a larger rectangle or a rectangular strip.

The problem of packing circles into a circular container was well studied in the literature. The best known packings for equal circles (up to $n = 1000$ with unit radius) are recorded on the website http://www.packomania.com maintained by Specht. Optimal solutions have been proven only for $n \leqslant 13$, and $n = 19$, and most researchers aim to find the best possible packing patterns by heuristic or meta-heuristic methods (Graham, Lubachevsky, Nurmela, & Östergård, 1998; Grosso, Jamali, Locatelli, & Schoen, 2010; Huang & Ye, 2011). The best known packing patterns for unequal circles on classic benchmarks are published by Akeb and Hifi (2008), Akeb and Hifi (2010), Akeb, Hifi, and Negre (2011), Huang, Li, Akeb, and Li (2005) and etc., and the methods include constructive heuristics, global search heuristics, or their combinations. For instance, Addis, Locatelli, and Schoen (2008) and Grosso et al. (2010) used a Monotonic Basin Hopping (MBH) approach to find the funnel bottom, and in order to tackle the difficulty on the existence of many funnels, they proposed a Population Basin Hopping strategy that maintained a pool of solutions and used a dissimilarity measure to guarantee the diversification. Inspired by a physical

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phenomenon on the movement of the elastic objects, Huang and Zhan (1982) proposed a quasi-physical approach to solve the circle packing problem. And then, Huang, Li, Li, and Xu (2006) proposed two new heuristic algorithms, called A1.0 and A1.5, to pack unequal circles into a two-dimensional circular container. A1.0 chose a feasible position having the highest hole degree for a circle to be packed, and A1.5 is a modified version of A1.0 applying a look ahead search for every feasible corner position. In 2011, Huang and Ye (2011) proposed a quasi-physical global optimization method by simulating two kinds of movements for the elastic disks: one was a smooth movement driven by elastic forces, the other was a abrupt movement driven by strong repulsive forces and attractive forces. Castillo, Kampas, and Pinter (2008) described different "pure" and industrial circle packing problems and gave a review on exact and heuristic methods. In order to make the searching space further diversified, Akeb, Hifi, and M'Hallah (2009) used an adaptive heuristic that combined a dichotomous beam search (BS) and used the notion of maximal hole degree (MHD) to find a position for a circle at each node of the BS tree.

For the second category, the two-dimensional circular open dimension problem (CODP), where the items are circles and the container is a strip or a rectangle, solving methods also include constructive stage and global search stage. Addis, Locatelli, and Schoen (2008) proposed a stochastic search algorithm basing on a conjecture that CODP possessed a so-called funneling landscape, a feature that is commonly found in molecular conformation problems. Akeb and Hifi (2008) proposed three heuristics for CODP: an open strip generation strategy, an exchange-order strategy to augment the first heuristic and a hybrid heuristic that combined a beam search with a series of predetermined interval search. For further improvements, Akeb et al. (2011) proposed an augmented algorithm which incorporated a beam search, a binary search, a multi-start strategy and a separate-beams strategy. Meanwhile, they also proposed an adaptive look-ahead strategy-based algorithm in Akeb and Hifi (2010). Huang et al. (2005) developed two greedy approaches denoted by B1.0 and B1.5 basing on the maximal hole degree. In order to improve B1.0 and B1.5, Kubach, Bortfeldt, and Gehring (2009) developed several greedy algorithms and parallelized them by a master slave approach followed by a subtree-distribution model. Fu, Huang, and Lü (2013) proposed an Iterated Tabu Search approach (ITS), which combined a tabu search procedure (TS) and a solution perturbation operator associated with an acceptance criterion.

This paper addresses an important extension of CPP, the circle packing problem with equilibrium constraints (CPPEC). It is a global optimization problem with the background of the layout design of satellite modules, space station, rotating building, multiple spindle box, etc. In recent years, researchers have reported several efficient approaches for solving CPPEC. Teng, Sun, Ge, and Zhong (1994) proposed a method of model-changing iteration and a method of main objects topo-models. And then, in 1999 Tang and Teng (1999) proposed a decimal coded adaptive genetic algorithm, which alleviated the combinatorial explosion and the premature convergence of the genetic process. Qian, Teng, and Sun (2001) proposed a human–computer interactive genetic algorithm that made the artificial individuals as a part of the chromosome population and replaced the worst individuals after copy, crossover, and mutation operations. Li, Liu, and Sun (2004) presented a mutation particle swarm optimization (PSO) algorithm by imposing a mutation operator on PSO in the later phase of the convergence. Based on the work in Wang, Huang, Zhang, and Xu (2002) and Huang and Chen (2006) proposed an improved version of the quasi-physical & quasi-human algorithm by introducing an efficient strategy to accelerate the searching process. Liu and Li (2010) proposed a basin filling algorithm by combining an improved energy landscape paving (ELP) method, a gradient method basing on local search and a heuristic configuration update mechanism. And in Liu, Li, Chen, Liu, and Wang (2010), they further developed a simulated annealing algorithm by incorporating the heuristic neighborhood search mechanism with the adaptive gradient method. In 2011, Liu, Li, and Geng (2011) proposed a heuristic algorithm with tabu search for solving CPPEC. The algorithm applied a gradient method with adaptive step length so as to find the minimum energy configuration and adopted the strategy of tabu search to jump out of the local minima. And the current best records have been maintained by Liu et al. (2011) since 2011.

Inspired by the coarse-to-fine control process in the manufacture industry, we propose a Coarse-to-Fine Quasi-Physical (CFQP) optimization method in this paper. Two new quasi-physical models are proposed to guide the system to local minima, and a basin hopping with tabu method are adopted to find new promising areas. Then, owing to the coarse-to-fine control strategy, CFQP neither spends too much searching time at local minimum traps nor easily misses promising areas that may lead to a global optimum. We also adapt the CFQP method to solve CPP. Computational results indicate the effectiveness of the proposed methods for CPPEC and CPP.

2. Problem definition

2.1. Problem description

Given n disks (n is a positive integer), with each disk i ($i \in \{1, 2, \ldots, n\}$) having radius R_i, mass m_i (R_i and m_i are positive real numbers), and given a small positive real number δ_r, the circle packing problem with equilibrium constraints (CPPEC) requires a dense packing of the n disks that highly concentrates on the center of a circular container, such that the radius of the container is as small as possible. A feasible layout is the one satisfying the following constraints:

(1) any one disk is totally in the container;
(2) any two disks do not overlap each other, namely, their intersection area is 0;
(3) the deviation distance between the centroid of the n disks and the center of the container is less than δ_r.

In the relevant aerospace applications, a layout of the disks corresponds to a cross-section of the satellite modules, and the center of the container corresponds to the axis of the satellite projection. In order to reduce the resistance, it is required that the radius of the satellite is as small as possible. Meanwhile, if the static non-equilibrium value of a spinning satellite is greater than 0, then the rotational speed of the modules cannot be uniform, leading to vibration, heat, noise, abrasion, etc. Therefore, constraint (3) requires that the deviation from the dynamic-balancing of the total system is less than a small positive real number, such that the resultant moment of the modules in a spinning satellite, namely the static non-equilibrium value, is less than $\delta_J = (\sum_{i=1}^{n} m_i) \cdot \delta_r$. For other applications in which there is no rotation or the high-speed rotation is not necessary, constraint (3) corresponds to the stability of the whole system, requiring that the deviation between the centroid of the system and the center of the container is as small as possible, referred to as the stability constraints in the literature.

2.2. Problem formulation

In a two-dimensional Cartesian coordinate system, let the center coordinate of the container be (0,0) and let the center coordinate of disk i ($i \in \{1, 2, \ldots, n\}$) be (x_i, y_i), as shown in Fig. 1. Then