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Robust single machine scheduling for minimizing total flow time in the presence of uncertain processing times

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ABSTRACT

This research deals with the single machine scheduling problem (SMSP) with uncertain job processing times. The single machine robust scheduling problem (SMRSP) aims to obtain robust job sequences with minimum worst-case total flow time. We describe uncertain processing times using intervals, and adopt an uncertainty set that incorporates a budget parameter to control the degree of conservatism. A revision of the uncertainty set is also proposed to address correlated uncertain processing times due to a number of common sources of uncertainty. A mixed integer linear program is developed for the SMRSP, where a linear program for determining the worst-case total flow time is integrated within the conventional integer program of the SMSP. To efficiently solve the SMRSP, a simple iterative improvement (SII) heuristic and a simulated annealing (SA) heuristic are developed. Experimental results demonstrate that the proposed SII and SA heuristics are effective and efficient in solving SMRSP with practical problem sizes.

1. Introduction

The single machine scheduling problem (SMSP) is one of the most studied production scheduling problems due to its complexity and practical importance. This problem aims at obtaining the best sequence for a set of jobs in a manufacturing system with a single machine. The total flow time (TFT), makespan, total tardiness, or their weighted combinations are performance measures frequently used in such a system. In an industrial setting, manufacturing systems usually operate in highly uncertain environments in which interruptions (mostly random in nature) prevent the execution of production schedules exactly as they were developed. Particularly, variation in processing times and other stochastic events (e.g., machine breakdowns, rush orders, order cancellations, and raw material shortages) lead to the variability in manufacturing systems (Sabuncuoglu & Goren, 2009).

This research addresses the SMSP with uncertain job processing times. Recognizing the need to take data uncertainty into account, researchers have adopted different methodologies to obtain robust schedules in the single machine context. Stochastic programming (SP) is a classical approach to tackling job data uncertainty in SMSPs (e.g., Agrawala, Coffman, Garey, & Tripathi, 1984; Birge & Louveaux, 1997; Soroush, 2007; Trietsch & Baker, 2008; van den Akker & Hoogeveen, 2008; Wu & Zhou, 2008). Although SP provides a theoretically sound foundation for SMSPs with random data, the application of SP to large size stochastic SMSPs is hindered due to the following reasons: (i) probabilistic distribution knowledge of input data is required, (ii) optimization of expectations may be suitable for (long term) planning purposes but not of practical interest from the operational viewpoint, and (iii) the problem size will increase exponentially with the number of uncertain parameters (see e.g., Ben-Tal & Nemirovski, 1999; Bertsimas & Sim, 2003; Daniels & Kouvelis, 1995).

Robust optimization (RO; e.g., Mulvey & Vanderbei, 1995; Kouvelis & Yu, 1997), which optimizes against the worst instances using a min–max objective, is another approach that has been employed to solve SMSPs with uncertain data. Daniels and Kouvelis (1995) generated robust schedules to minimize the TFT for the SMSP with uncertain processing times. They defined schedule robustness according to absolute or relative deviations from optimal TFT in the worst-case scenario, and showed that the discrete-scenario version of the problem is NP-hard. A branch-and-bound exact algorithm and two surrogate relaxation heuristics were also developed in their research to obtain robust schedules. Yang and Yu (2002) dealt with the SMSP with uncertain processing times represented as a discrete set of scenarios, and presented an exact dynamic programming algorithm and two heuristics to obtain robust schedules. Kasperski (2005) considered uncertain

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due dates and processing times represented as interval data in an SMSP with precedence constraints, and developed a polynomial time algorithm for constructing robust schedules with minimum worst-case deviation from optimal performance. Lebedev and Averbakh (2006) addressed a robust SMSP, in which the processing time of each job is represented by a prescribed interval or range. They proved that the problem is NP-hard when the min–max regret criterion is used, and showed that the problem can be solved in $O(n\log n)$ time if the number of jobs is even and all intervals of uncertainty have the same center. Montemanni (2007) presented a mixed integer linear programming (MILP) formulation for the robust SMSP, introduced by Lebedev and Averbakh (2006), and discussed some preprocessing rules which can be utilized to efficiently solve the MILP formulation. Kasperski and Zieliński (2008) considered the TFT-minimization SMSP with processing times represented as intervals and job precedence constraints. They showed that this problem is approximable within 2 if its deterministic counterpart is polynomially solvable under a mid-point scenario. The above excellent references significantly reduce the gap between theoretical progresses and industrial practices for the SMSP with uncertain data.

With increasing awareness of the importance of solution robustness, this past decade witnessed continuous development of RO-related techniques and renewed interest in robust counterpart optimization (RCO) approaches to dealing with data uncertainty in mathematical programming. Early in the 1970s, Soyster (1973) considered coefficient uncertainty in linear programming (LP) formulations and proposed an equivalent LP model that admits the highest protection by designating all uncertain parameters to take their boundary values simultaneously. As a result, this approach may be too conservative in practice and the robust solution is much worse than the optimal solution of the nominal problem. To address the problem of over-conservativeness, El-Ghaoui, Oustry, and Lebret (1998) and Ben-Tal and Nemirovski (1999, 2000) developed the RCO formulations of LP and convex programming problems, which assume that uncertainty sets of data are ellipsoids. These formulations have the flexibility to control the degree of solution conservatism through safety parameters and constraint violation probabilities. Bertsimas and Sim (2004) have recently proposed a different RCO approach that reformulates nonlinear robust models as linear models according to the duality theory of LP. Bertsimas and Sim (2003) demonstrated that this RCO approach can be directly applied to discrete optimization models and network flow problems.

This research addressed the TFT-minimization SMSP with uncertain job processing times represented by intervals. This single machine robust scheduling problem (SMRSP) aims to obtain robust job sequences with minimum worst-case TFT. This min–max criterion is different from the min–max regret criterion used by Lebedev and Averbakh (2006). In this research the uncertainty set, proposed by Bertsimas and Sim (2003, 2004), is adopted for interval-represented processing times. Not only does it include a budget parameter for controlling the number of coefficients that can simultaneously take their largest variations, it also provides a way of incorporating different attitudes toward risk (e.g., risk-averse, risk neutral, or risk-seeking). Hence, the decision maker can select the job schedule which achieves a balance between robustness and optimality.

While most previous RO/RCO models assume independently distributed uncertain parameters, uncertain job processing times may be attributed to several *common* sources of uncertainty (e.g., shortage of raw materials, machine breakdowns, and employee sick leave), and hence correlated, in some manufacturing systems. A revision of the uncertainty set is proposed to address correlated uncertain processing times due to a number of common sources of uncertainty. The case in which an uncertainty source has distinct levels of impact on different groups of uncertain data is also addressed. For instance, jobs are often classified into a number of groups (e.g., Webster & Baker, 1995), each of which may require additional resources (e.g., raw materials, equipments, or labor skills), and hence the unavailability of one particular resource may only have impact on the job group that is highly dependent on that resource but no (or very little) impact on other job groups.

This research develops a mixed integer linear programming (MILP) model for the SMRSP. The basic idea is to integrate a linear program developed to determine the worst-case TFT within the integer program of the (deterministic) TFT-minimization SMSP. To efficiently solve the SMRSP, a simple iterative improvement (SII) heuristic and a simulated annealing (SA) heuristic are developed. Numerical experiments are conducted to demonstrate the effectiveness and efficiency of SII and SA heuristics. The tradeoff between robustness and optimality is examined and the impact of the degree of uncertainty on the performance measures is also explored.

The remainder of this paper is organized as follows. Section 2 defines the SMRSP of interest and presents the MILP model for the SMRSP. The SMRSP with correlated data uncertainty is also addressed and the revised uncertainty set for this problem is defined. Section 3 describes the proposed SII and SA heuristics. Numerical experimental results are provided in Section 4, followed by concluding remarks given in Section 5.

2. Problem statement and models

2.1. Single machine robust scheduling problem (SMRSP)

Consider a set N of n jobs to be processed on a single machine with the processing time of job j being p_j, $j = 1, \ldots, n$. All jobs are released at the beginning of the scheduling period. A classical SMSP in the literature is to find an optimal processing sequence of the jobs which minimizes the sum of completion times (or TFT) of all jobs. Let $x_{ij} = 1$ if job j is assigned to the ith position of the sequence and 0 otherwise. $\boldsymbol{x} = \{x_{ij}, i, j = 1, \ldots, n\}$. The integer programming (IP) formulation of the SMSP is as follows.

$$\text{(SMSP) Minimize } C \tag{1}$$

$$\text{Subject to } C \geqslant \sum_{j=1}^{n}\sum_{i=1}^{n}(n-i+1)p_j x_{ij}, \tag{2}$$

$$\sum_{i=1}^{n} x_{ij} = 1, \qquad j = 1, \ldots, n, \tag{3}$$

$$\sum_{j=1}^{n} x_{ij} = 1, \qquad i = 1, \ldots, n, \tag{4}$$

$$x_{ij} \in \{0,1\}, i = 1, \ldots, n, \qquad j = 1, \ldots, n. \tag{5}$$

It is well known that an optimal solution $\boldsymbol{x}^*$ to the above SMSP can be obtained using the classical shortest processing time (SPT) rule. However, particularly when job processing times are increased due to unexpected interruptions, this classical approach, which determines job schedules according to nominal processing times, may lead to substantially unsatisfactory scheduling performance. Thus, a robust scheduling approach has to explicitly take into account variations in job processing times that result in significant delays to production schedules.

In this research, uncertain processing times (i.e., the increase of processing times due to interruptions) are represented using the following uncertainty set:

$$U_1 = \left\{ \boldsymbol{p} | p_j \in [\bar{p}_j,\ \bar{p}_j + \hat{p}_j],\ \forall j \in N; \sum_{j \in N} \frac{p_j - \bar{p}_j}{\hat{p}_j} \leqslant \Gamma \right\},$$