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Joint replenishment with imperfect items and price discount ☆

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ABSTRACT

A joint replenishment problem is presented to determine the ordering policy for multiple items having a certain percentage of defective units. The purpose of this paper is to study the impact of the percentage of defective units on the ordering policy. Two different scenarios are presented for joint replenishment problem: (1) without price discount and (2) with price discount. For each scenario, the total expected cost per unit time is derived and algorithms are presented to determine the family cycle length and the integer number of intervals that the replenishment quantity of each item will last. Numerical examples are presented and the results are discussed.

1. Introduction

In a multi-item inventory system, a joint replenishment policy is defined as the coordination of the replenishment of a group of items that may be ordered jointly from a single supplier. On the buyer's side, by sharing the fixed cost associated with the joint replenishment of a group of items, the total inventory cost can be reduced by the joint replenishment policy, compared to the individual and independent replenishment policy. Similarly, on the supplier's side, the set-up cost can be saved by manufacturing multiple items using the same facility. A joint replenishment policy is also a good practice to reduce transportation cost, if a vehicle is used to simultaneously transport the jointly ordered multiple items. Therefore, the joint replenishment policy leads to a lower unit cost, lower transportation cost, lower ordering cost, lower setup cost, lower purchasing cost, and easy scheduling. On the other hand, the joint replenishment policy increases the average inventory level and system control costs but reduces the flexibility since multiple items are stocked at a single point.

In the literature, there are many studies that deal with joint replenishment problem. At the early stage, Shu (1971) proposed a simple procedure to determine the order quantity for a joint replenishment problem. Then, Goyal (1974) introduced an enumeration approach and it was claimed that this approach always yields the optimal solution. However, in order to reduce complexity in solving joint replenishment problem, Silver (1975) proposed a non-iterative method. Then, van Eijis, Heuts, and Kleijnen (1992) made a comparison between two different types of strategies for a joint replenishment problem: one is direct grouping strategy and the other is indirect grouping strategy. In direct grouping strategy, the integer number of intervals that the replenishment quantity of an item will last is not an integer multiple of the family cycle time. However, in indirect grouping strategy, the integer number of intervals that the replenishment quantity of an item will last is an integer multiple of the family cycle time. After measuring the performance of these two strategies, according to percentage of the total cost saving, the authors recommended the indirect grouping strategy. Hariga (1994) proposed two heuristic procedures that are able to give global optimal replenishment policies. Then, he also compared the proposed heuristic procedures with existing ones and showed that the proposed heuristic procedures can result in the lowest cost. Wildeman, Frenk, and Dekker (1997) presented an efficient optimal solution by applying Lipschitz optimization for a joint replenishment problem instead of using a heuristic solution. A genetic algorithm model was proposed by Khouja, Michalewicz, and Satoskar (2000) and it was compared with RAND heuristic algorithm. Cha and Moon (2005) presented a modified RAND heuristic algorithm and a modified genetic algorithm. Recently, Hong and Kim (2009) proposed a new genetic algorithm by estimating the exact average cost. Yoo and Lin (2009) developed another algorithm where genetic algorithm and simulated annealing are cooperatively used to reduce the total relevant cost.

The above studies in the literature on the joint replenishment policy assume that all units in all items procured from a supplier have perfect quality and no defects. However, Wahab, Mamun, and Ongkunaruk (2011) indicated that although many small and

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medium enterprises implemented total quality management to ensure the quality of the units manufactured, buyers still find defective units, which fail to meet the quality requirements defined by buyers, in the procured items. There are a number of studies that investigated the effect of defective units on the economic order quantity (EOQ) inventory model. For example, Salameh and Jaber (2000) developed an EOQ model where each shipment consisted of a percentage of defective units, which were sold at a discount price after completely screening the lot. For the same model, a simple solution procedure was presented in Goyal and Cardenas-Barron (2002). Then, Papachristos and Konstantaras (2006) derived the sufficient condition to ensure no shortages at the end of the cycle. Maddah and Jaber (2008) revisited and corrected the flaw in Salameh and Jaber (2000). Wahab and Jaber (2010) considered different holding cost for good and defective items for the model presented in Salameh and Jaber (2000). Recently, Khan, Jaber, Guiffrida, and Zolfaghari (2011) presented a comprehensive review on EOQ models with defective units, and Jaber, Zanoni, and Zavanella (2013) proposed an EOQ model with imperfect quality based on entropy. However, to the authors' knowledge there is no study considering the presence of defective units in a joint replenishment policy.

This paper contributes to the literature in the area of joint replenishment problem. In particular, a joint replenishment policy is developed when the shipment size for each item has a percentage of defective items.

This paper proposes an ordering policy for multiple items, where each item has a certain percentage of defective units in each shipment. A joint replenishment model is considered to determine the family cycle length and the integer number of family cycles that the replenishment quantity of an item will last by minimizing the total cost associated with the system. The demand for each item is considered to be deterministic and shortages are not allowed; and therefore each item will be replenished when its inventory level drops to the zero level. Since each item has a percentage of defective units in each replenishment, the replenished lot is screened by the buyer to separate the defective units while only the non-defective units satisfy the demand. When the screening is completed, the segregated defective units in each item are disposed at a certain cost. Two different models are proposed: first, a joint replenishment model with no price discount is introduced; second, another model is introduced with a group discount on unit price, where the buyer will receive a price discount when the total replenishment quantity exceeds the breakpoint quantity.

This paper is organized as follows: Section 2 presents the model for a joint replenishment problem with defective units and provide an algorithm to determine decision variables. Section 3 presents a joint replenishment problem with price discount and an algorithm to determine decision variables. Section 4 illustrates numerical examples. Section 5 concludes the paper.

2. Joint replenishment with imperfect quality

Item $j \in \{1, 2, \ldots, n\}$ has demand D_j, which is assumed to be deterministic and constant; shortages are not allowed; and the initial inventory is zero. An order for a family of items is placed at a time. However, all n items may not be ordered in each replenishment. Item j is ordered in every m_j^{th} replenishment, where m_j is the integer number of T intervals that the replenishment quantity of item j will last. This means that the cycle time for item j is m_jT. It is assumed that p_j is the percentage of defective for item j in each replenishment, where p_j is a random variable. Once the replenishment is received, the lot for each item is screened at the rate of $x_j (> D_j)$ during time t_j, where $t_j = Q_j/x_j$, where Q_j is the order quantity for item j. An example of the inventory profile for three items is given in Fig. 1. For item j, the expected number

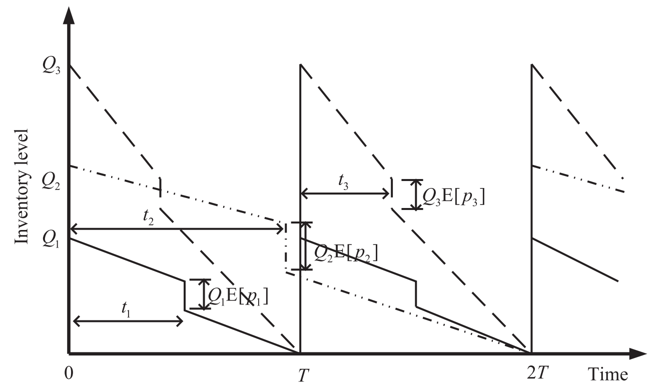

Fig. 1. Inventory profile for three items.

of non-defective units in each replenishment is given by $Q_j(1 - \mathbb{E}[p_j])$, and only non-defective units will satisfy the demand in each cycle. Hence, in order to ensure no shortages, the following condition must be satisfied:

$$Q_j(1 - \mathbb{E}[p_j]) \geqslant D_j t_j. \tag{1}$$

Substituting the value of t_i, it can be expressed as

$$\mathbb{E}[p_j] \leqslant 1 - \frac{D_j}{x_j}. \tag{2}$$

Since m_j is the integer number of T intervals that the replenishment quantity of item j will last, the replenishment quantity Q_j for item j is given by

$$Q_j(1 - \mathbb{E}[p_j]) = D_j m_j T, \tag{3}$$

where Q_j consists of non-defective and defective units. The average inventory level of the non-defective and defective units is given by

$$I_j = \frac{Q_j(1 - \mathbb{E}[p_j])}{2} + \frac{Q_j \mathbb{E}[p_j] t_j}{m_j T}. \tag{4}$$

By substituting Q_j and t_j, the above equation can be expressed as follows:

$$I_j = \frac{D_j m_j T}{2} + \frac{D_j^2 m_j T \mathbb{E}[p_j]}{x_j (1 - \mathbb{E}[p_j])^2}. \tag{5}$$

The total cost function of the joint replenishment with defective units is a summation of major ordering cost associated with a replenishment of the family, minor ordering cost when a particular item is ordered, unit purchasing cost, screening cost, inventory holding cost, and salvage cost of defective units.

A family or joint ordering cost K is incurred every T units of time, which is independent of the composition of the order quantity. Whereas a minor ordering cost a_j for item j is incurred only once in every m_jT unit of time. Therefore, the total ordering cost per unit time is

$$\frac{K + \sum_{j=1}^{n} \frac{a_j}{m_j}}{T}. \tag{6}$$

A holding cost h_j for item j is incurred per unit time. Holding cost, $h_j = ic_j$, where i is the interest rate and c_j is the unit purchasing cost. Therefore, the total expected holding cost per unit time is

$$\sum_{j=1}^{n} h_j \left[\frac{D_j m_j T}{2} + \frac{D_j^2 m_j T \mathbb{E}[p_j]}{x_j (1 - \mathbb{E}[p_j])^2} \right], \tag{7}$$

and the total purchasing cost per unit time is

$$\sum_{j=1}^{n} \frac{c_j D_j}{1 - \mathbb{E}[p_j]}. \tag{8}$$