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Reliability-based robust design optimization: A general methodology using genetic algorithm

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ABSTRACT

In this paper, we present an improved general methodology including four stages to design robust and reliable products under uncertainties. First, as the formulation stage, we consider reliability and robustness simultaneously to propose the new formulation of reliability-based robust design optimization (RBRDO) problems. In order to generate reliable and robust Pareto-optimal solutions, the combination of genetic algorithm with reliability assessment loop based on the performance measure approach is applied as the second stage. Next, we develop two criteria to select a solution from obtained Pareto-optimal set to achieve the best possible implementation. Finally, the result verification is performed with Monte Carlo Simulations and also the quality improvement during manufacturing process is considered by identifying and controlling the critical variables. The effectiveness and applicability of this new proposed methodology is demonstrated through a case study.

1. Introduction

Quality and reliability improvement through variation reduction can prepare a solid ground for manufacturing and service processes, leading to a competitive advantage. In order to achieve a desirable and reliable product, we need to consider it necessary to improve the system performance. The majority of quality improvement attempts concentrate on improving quality during manufacturing process. However, it is obvious that when quality and reliability are designed for a product, the product performance improves significantly. An effective approach is to use optimization techniques in order to determine the optimal design settings to reduce the costs while maintaining the desired performance. Although deterministic optimization techniques have been used successfully in many areas of the engineering design, undesired results should be expected when uncertainties in design variables and/or modeling parameters are not taken into account. As we approach the limits of one or more of the design settings in deterministic design optimization (DDO) approach, design optimality becomes unreliable, leading to product failure. In recent years, different approaches have been developed to investigate the effect of uncertainties. Among these approaches, reliability-based design optimization (RBDO) (see for example, Fang, Gao, Sun, & Li, 2013; Sinha, 2007; Valdebenito & Schuëller, 2011; Yao, Chen, Huang, & Tooren, 2013; Youn, Choi, Yang, & Gu, 2004) and robust design optimization (RDO) approaches (Beyer & Sendhoff, 2007; Shin, Samanlioglu, Cho, & Wiecek, 2011; Taguchi, 1993) are effective tools to incorporate uncertainties in the design. RBDO achieves the target confidence of product reliability, while RDO minimizes the quality loss of the product.

Since the individual application of RDO and RBDO does not ensure the quality and reliability of a product during its life cycle, their concepts are combined in RBRDO (see for example, Du, Sudjianto, & Chen, 2004; Lee, Choi, Du, & Gorsich, 2008; Rathod, Yadav, Rathore, & Jain, 2013; Yadav, Bhamare, & Rathore, 2010; Youn, Choi, & Yi, 2005). We use this integration to propose a general methodology to design robust and reliable products effectively. Our methodology is composed of four stages: formulation, optimization, selection, and evaluation. In the first two stages, the problem is formulated based on RBRDO to simultaneously consider the reliability and robustness and also genetic algorithm (GA) is used to find the reliable and robust Pareto-optimal solutions. Next, the best solutions are chosen for implementation using some defined criteria. Then, the results are evaluated using Monte Carlo simulations (MCS) and critical variables, which should be controlled during manufacturing process, are identified.

Baril, Yacout, and Clément (2011) proposed a methodology to design robust products under some limitations. First, the use of worst-case analysis, which assumes that all changes may simultaneously occur in the worst possible combinations, has some pitfalls (Du & Chen, 2000). Second, it is required to simultaneously

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optimize the mean and the variance of the performance functions (quality characteristics) in the objective functions. Furthermore, when more than one performance function is used, the corresponding correlation structure should be considered. Our improved methodology overcomes the pitfalls of the available one. Worst-case analysis is replaced with probabilistic feasibility formulation. We consider both mean and variation of performance functions in the objective function. Moreover, the presence of more than one quality characteristic and the corresponding correlation are considered using mean-squared error (MSE) as the objective function.

Section 2 presents design optimization approaches under uncertainties and different methods considered to solve multi-objective optimization problems. Our general methodology is explained in Section 3. Section 4 presents an applicable example solved using the proposed methodology. Our concluding remarks are provided in the final section.

2. Literature review

2.1. Design optimization approaches under uncertainties

This section provides an overview of the approaches commonly used for design optimization under uncertainty.

2.1.1. Robust design optimization

Robust design proposed by Taguchi (1993) can help one to improve the quality of a product by optimizing the mean and minimizing the performance variance. A product designed using robust design concept should be insensitive to uncertainties. The review paper by Park and Lee (2006) gives a thorough discussion of robust design. In RDO, the robustness concept is added to conventional optimization problem that can be formulated as follows:

$$\begin{cases} \text{Min } f(\mathbf{d}) \\ S.t: g_j(\mathbf{d}) \leqslant 0 \quad j = 1, \ldots, J \\ \mathbf{d}^{\mathrm{L}} \leqslant \mathbf{d} \leqslant \mathbf{d}^{\mathrm{U}} \end{cases} \tag{1}$$

where $\mathbf{d}$, $\mathbf{d}^{\mathrm{L}}$, $\mathbf{d}^{\mathrm{U}}$ are the vectors of deterministic design variables, their lower and upper bounds, respectively, f is the performance function, and g_j is the jth constraint function. When uncertainties in design variables and design parameters are considered, the objective and constraint functions are modified as $f(\mathbf{d}) \to f(\mathbf{d}, \mathbf{X}, \mathbf{P})$, $g_j(\mathbf{d}) \to g_j(\mathbf{d}, \mathbf{X}, \mathbf{P})$ and then, RDO is stated as follows (Park & Lee, 2006):

$$\begin{cases} \text{Min } \acute{f}(\mathbf{d}, \mathbf{X}, \mathbf{P}) \\ S.t: \acute{g}_j(\mathbf{d}, \mathbf{X}, \mathbf{P}) \leqslant 0 \quad j = 1, \ldots, J \\ \mathbf{d}^{\mathrm{L}} \leqslant \mathbf{d} \leqslant \mathbf{d}^{\mathrm{U}} \end{cases} \tag{2}$$

In the above model, $\acute{f}$, the objective function and $\acute{g}_{j,}$ the jth constraint function, are functions of the probabilistic characteristics of the modified performance function f and g, respectively. The probabilistic characteristic could include the mean, the standard deviation or the combination of both. $\mathbf{d}$ and $\mathbf{X}$ are the vectors of deterministic and random design variables, respectively. $\mathbf{P}$ is the vector of random design parameters. A design variable (whether deterministic or random) is changeable and controllable by a designer in a design process while a design parameter is not. The decision variables are $\mathbf{d}$ and the mean of random design variables $\mathbf{X}$. In this paper, bold letters are used for vectors, capital letters for random variables and parameters, and lower case letters for deterministic variables.

The main concern of RDO is how accurately and efficiently the statistical moments of the performance function can be estimated. The commonly used methods to evaluate these moments are numerical integration, analytical methods, and simulation methods. Because of their shortcomings, other approximation methods such as univariate dimension reduction method (DRM), performance moment integration (PMI), and percentile difference method (PDM) have been recently proposed to estimate the moments. Lee et al. (2008) investigated the efficiency and accuracy of DRM, PMI, and PDM methods. Moreover, Youn et al. (2005) used PMI in some examples to determine statistical moments and its efficiency had been shown for even non-normal distributed statistical responses with high skewness and kurtosis. In this paper, PMI will be used to determine the first and second statistical moments.

Design feasibility in the presence of variability and uncertainty is another concern in RDO. Different approaches to consider design feasibility under uncertainties are classified into methods which require probability and statistical analyses and methods that do not (Du & Chen, 2000). The former methods, which ensure that the solution achieves an accurate level of constraint satisfaction, are the best way to describe the feasibility robustness, although evaluating the probability of constraint satisfaction may have some computational difficulties. The following section provides the detailed discussion on RBDO in which the probabilistic feasibility formulation is used to consider uncertainties in the design variables and parameters.

2.1.2. Reliability-based design optimization

RBDO is a method to obtain optimal designs where the probability of failure is low. If $g_j(\mathbf{d}) \leqslant c$ is considered to be the jth deterministic constraint, the probabilistic feasibility formulation in the presence of uncertainty (random variables $\mathbf{X}$ and random parameters $\mathbf{P}$) is shown as follows.

$$P[G_j(\mathbf{d}, \mathbf{X}, \mathbf{P}) \leqslant 0] \geqslant R_j \quad j = 1, \ldots, J \tag{3}$$

where $G_j(\mathbf{d}, \mathbf{X}, \mathbf{P}) = g_j(\mathbf{d}, \mathbf{X}, \mathbf{P}) - c$ is the jth constraint function. c and R_j are the limiting value and the probability of satisfying this constraint (reliability level), respectively.

When the statistical distribution of all random design variables and parameters is known, the probability is computed based on the following integral:

$$P[G_j(\mathbf{d}, \mathbf{X}, \mathbf{P}) \leqslant 0] = \int \cdots \int_{G_j(\mathbf{d}, \mathbf{X}, \mathbf{P}) \leqslant 0} f_{\mathbf{X}, \mathbf{P}}(\mathbf{x}, \mathbf{p}) \mathrm{d}\mathbf{x} \mathrm{d}\mathbf{p} \tag{4}$$

where $f_{\mathbf{X},\mathbf{P}}(\mathbf{x}, \mathbf{p})$ is a joint probability density function of $\mathbf{X}$ and $\mathbf{P}$. However, most of the time it is practically difficult or impossible to calculate the numerical solution for the above equation. Hence, approximate procedures have been proposed to estimate this probability accurately and efficiently. One alternative procedure is to use sampling methods such as MCS, Latin hypercube sampling, importance sampling, most probable point (MPP) based importance sampling or directional sampling (Dubourg, Sudret, & Deheeger, 2013; Padmanabhan, Agarwal, Renaud, & Batill, 2006; Rashki, Miri, & Azhdary Moghaddam, 2012). However, when the desired probability of failure ($1 - Rj$) is low, the computational effort of sampling methods is prohibitively expensive (Du & Chen, 2001). Another alternative procedure is optimization-based methods (e.g. first and second order reliability methods (FORM/SORM)) which are based on linear (FORM) or quadratic (SORM) approximation of the constraint boundary at an MPP, to ensure minimal accuracy loss (Agarwal, 2004).

The main concept of the later class of reliability assessment methods is to determine MPP which was first introduced in the structural analysis (Hasofer & Lind, 1974). The first step to compute the MPP is to transform the original random variables and parameters ($\mathbf{X}$, $\mathbf{P}$) in original space into standard normal variables and parameters ($\mathbf{U}_{\mathbf{X}}$, $\mathbf{U}_{\mathbf{P}}$) in $\mathbf{U}$-space (the independent and standardized normal space) using Rosenblatt transformation