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Technical Note

A learning curve for tasks with cognitive and motor elements

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ABSTRACT

This paper develops a new learning curve model that has cognitive and motor components. The developed model is fitted to experimental data of a repetitive manual assembly-and-disassembly task. The fits are compared to those of two other known models from the literature, which are the renowned power form learning curve and its aggregated version. The model developed in this paper performed the best. The fits of the models are evaluated using the mean squared error method. Furthermore, the developed learning curve model is investigated by incorporating it into the economic production quantity model, a topic which has been frequently studied by researchers. The results show that assuming an inappropriate learning curve may produce biased inventory policies by over- or underestimating production rates and consequently inventory levels.

1. Introduction

Learning curves have been valuable management tools for decades. They predict and monitor the performance of individuals, groups of individuals and organizations. They have been widely used and applied in various sectors (manufacturing, healthcare, energy, military, information technologies, education, design, banking, and more). There are various forms of learning curves that are available in the literature (see for instance, Yelle, 1979; Hackett, 1983; Badiru, 1992; Badiru & Ijaduola, 2009; Jaber, 2006, chap. 32). All these models suggest that performance improves with repetition. Readers may refer to Jaber (2011) for theory, models and applications of learning curves.

Of all the available models, the Wright (1936) learning curve remains to be the most popular (e.g., Yelle, 1979; Globerson, 1980; Badiru, 1992; Jaber, 2006, chap. 32). Its popularity is attributed to its simple mathematics and to its ability to fit a wide range of data fairly well (see Lieberman, 1987). Despite its popularity, the Wright learning curve, which is of a power form, has been criticized. For example, the results obtained from the Wright learning curve are not meaningful as the cumulative production approaches infinity. De Jong (1957) suggested introducing a plateauing factor to resolve this issue. Dar-El, Ayas, and Gilad (1995), based on evidence from the psychology and industrial engineering literature, suggested that the Wright learning curve is an aggregate learning curve that captures the cognitive and motor elements of a task or an experiment. To address this limitation, Dar-El et al. (1995) proposed the dual-phase learning curve model (DPLCM). Another limitation is that the Wright learning curve assumes that all units produced conform to quality. This assumption is unrealistic as many production processes are imperfect producing defective items that need to be reworked. Jaber and Guiffrida (2004) proposed a composite learning curve that is the sum of two learning curves; one describes the reduction in time for each additional unit produced while the other describes the reduction in time for each additional defective unit reworked. The composite learning curve model was found to have three behavioral patterns: Convex, plateau, and continuously decreasing. Only, the last behavior conformed with that of Wright (1936).

Along the same line of research, this paper proposes a composite learning curve model that is similar to the dual-phase learning curve model. The developed model is fitted to experimental data taken from the study of Bailey (1989) and its fits are compared to those produced from the Wright learning curve and the learning curve of Dar-El et al. (1995). The data used in this study was collected in a laboratory experiment of a repetitive task that involved assembling and disassembling a mechanical apparatus performed by paid subjects. The assembly task was described by the author as being more complex than the disassembly task. In the course of Bailey's study, workers were trained for 4–8 h in assembling and disassembling the apparatus. After a break of up to 4 months, the assembly-and-disassembly task was continued to analyze whether the subjects had forgotten some of the previously acquired skills. The assembly and disassembly process was overlooked and recorded by the researcher. For further information on the study, the reader is referred to the paper of Bailey (1989). In this study, the data of the learning sessions were used; i.e., the

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data from the relearning (second) sessions were ignored as this study does not account for forgetting effects. This is beyond the scope of the paper and would be considered in a future work.

The effects of learning on the economic order/production quantity (EOQ/EPQ) model (or the lot sizing problem) have been investigated frequently in the literature (e.g., Jaber & Bonney, 1999; Jaber & Bonney, 2011). Earlier studies assumed that learning in production may significantly reduce the total cost of an inventory system if a policy of producing smaller lots more frequently is adopted. Later studies investigated the combined or individual effects of learning in production, setups and quality on the lot sizing problem (Jaber & Bonney, 2003) or studied how learning influences the supplier selection decision (Glock, 2012), for example. The importance of learning in modern manufacturing and its effects on inventory policies have led some researchers to study its effects in a wider context; e.g., multistage production systems, supply chains and reverse logistics. Knowing how humans learn in production systems and how learning affects the performance of the production process is important for several reasons. For example, this enables production planners to assess how inventory develops over time, which is important to avoid bottlenecks in a production system. Further, knowing how much production capacity is available over time helps in drafting work plans and supporting the decision of whether peaks in demand need to be balanced by employing contractual (temporary) workers. Reader may also refer to Jaber (2011) for the importance of using the learning curve to investigate industrial engineering problems. We chose to investigate the applicability of the developed learning curve model in the context of the lot sizing problem. The next section provides a brief introduction of the Wright and Dar-El et al. learning curves.

2. The learning curves of Wright (1936) and Dar-El et al. (1995)

The Wright (1936) learning curve is the earliest and the most popular model that depicts performance as a function of output. In addition, we note that the learning curve data used in this study is that of Bailey (1989), who found that the log-linear function fitted his data well, which is equivalent in fit results to the power-form curve of Wright (1936), and which is of the form $\log T_n = \log T_1 - b\log n$. The Wright model advocates that performance reduces by a constant percentage each time the cumulative number of repetitions doubles. The Wright learning curve is of the form

$$T_n = T_1 n^{-b}, \tag{1}$$

where T_n is the time to produce the nth unit, T_1 is the time to produce the first unit, n is the cumulative number of repetitions (units), and b is the learning exponent. The learning exponent is calculated as $b = -\log(\phi)/\log(2)$, where ϕ is the learning rate and it is a percentage between 100% and 50% corresponding, respectively, to $b = 0$ and $b = 1$. For example, if $T_1 = 10$ and the $\phi = 0.8$ (or 80%), then the time to produce the second, fourth, and eighth units are respectively $T_2 = 10 \times 2^{-0.3219} = 8$, $T_4 = 6.4$, and $T_8 = 5.12$.

Dar-El et al. (1995) proposed an aggregate form of the Wright learning curve, the dual-phase learning curve model, where the resultant curve is the sum of two – cognitive and motor – learning curves. They assumed that both curves are of power forms like Wright's. The dual-phase learning curve model (DPLCM) is of the form

$$T_n = (T_1^c + T_1^m)n^{-b^*} = T_1^c n^{-b_c} + T_1^m n^{-b_m}, \tag{2}$$

where T_1^c is the time to perform the first repetition under pure cognitive conditions, T_1^m is the time to perform the first repetition under pure motor conditions, b_c is the learning exponent under pure cognitive conditions, and b_m is the learning exponent under pure motor conditions. The terms T_n and n were defined earlier. The learning exponent as observed after n repetitions was given as

$$b^* = b(n) = b_c - \frac{\log((R + n^{b_c - b_m})/(R+1))}{\log(n)}, \tag{3}$$

where $R = T_1^c/T_1^m$. In Dar-El et al. (1995), based on empirical data, the values of the learning exponents were taken as $b_c = 0.514$ ($\phi_c = 70\%$) and $b_m = 0.152$ ($\phi_m = 90\%$). However, they noted that further research could show that the values of the cognitive and motor exponents may differ from 0.514 and 0.152, respectively.

3. The proposed learning curve model

It is assumed that a worker performing a task (e.g., assembling an item) will refer to procedure or steps prior to and during the execution of the task (e.g., looking up some information in a manual). That is, a portion of the time to perform each task will be to process information and acquire knowledge necessary to perform the task. For example, a worker operating on an assembly line where customized products are produced may have to refer to a manual or process description each time a product variant arrives at the workstation to look up how the production steps need to be performed. Similarly, in a job shop production process, workers may need to refer to manuals when changing from one job to the next to look up information on how to process the items in question. We refer to this process step as build-up of knowledge. After the worker has looked up the necessary information and read it in the manual, i.e. after knowledge has been built up, the production steps are performed. This process is referred to as the knowledge retrieval step. It is intuitively clear that learning may occur in both process steps: after the worker has produced product variants several times, it requires the worker less time to look up information about the production process, and the production process itself might as well be performed faster. The first effect is commonly described as cognitive learning, whereas the second effect is termed motor learning.

The model proposed here is similar in form to that of Dar-El et al. (1995), and it is of the form

$$T_n = xT_1 n^{-b_c} + (1-x)T_1 n^{-b_m} = T_1[x(n^{-b_c} - n^{-b_m}) + n^{-b_m}], \tag{4}$$

where x is a percentage of splitting T_1 into two components, cognitive and motor; i.e. $T_1^c = xT_1$ and $T_1^m = (1-x)T_1$. The cognitive component of T_i for repetition $i \in [1, n]$ reduces at a faster rate than the motor component. This is logical as a worker tends to recall a procedure or steps faster with every repetition, perhaps to the extent of instant recall, whereas the motor component of T_i, T_i^m, is much larger than the cognitive component, T_i^c, and possibly restricted to a lower bound. The reader may wonder if Eqs. (2) and (4) are the same, but they are not. Eq. (2) does not provide a mechanism of how T_1 should be split between the motor and the cognitive component, while x in Eq. (4) is a model parameter that takes account of this fact. A second difference is that b_m and b_c in Eq. (2), or the DPLCM, are inputs and of fixed values, whereas b_m and b_c in JGLCM (Jaber-Glock learning curve model; Eq. (4)) are variables that are determined jointly with x to improve the model's fit to data.

4. The fits

In this section, the models presented in Section 2, WLC (Wright, 1936) and DPLCM (Dar-El et al., 1995), and the one presented in Section 3, which we will refer to as the JGLCM, are fitted to the experimental data of Bailey (1989). The experiment conducted by Bailey (1989) consisted of performing assembly tasks in a laboratory setting with students as surrogate workers in a manufacturing setting. Data for 102 learning curves were made available to us. To