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Two-machine no-wait flowshop scheduling with learning effect and convex resource-dependent processing times ☆

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ABSTRACT

Two-machine no-wait flowshop scheduling problems in which the processing time of a job is a function of its position in the sequence and its resource allocation are considered in the study. The primary objective is to find the optimal sequence of jobs and the optimal resource allocation separately. Here we propose two separate models: minimizing a cost function of makespan, total completion time, total absolute differences in completion times and total resource cost; minimizing a cost function of makespan, total waiting time, total absolute differences in waiting times and total resource cost. Since each model is strongly NP-hard, we solve both models by breaking them down to two sub-problems, the optimal resource allocation problem for any job sequence and the optimal sequence problem with its optimal resource allocation. Specially, we transform the second sub-problem into the minimum of the bipartite graph optimal matching problem (NP-hard), and solve it by using the classic KM (Kuhn–Munkres) algorithm. The solutions of the two sub-problems demonstrate that the target problems remain polynomial solvable under the proposed model.

1. Introduction

In classical scheduling theory, the job processing times are assumed as fixed and constant values (Pinedo, 2002). However, in numerous real-life situations, it is not rare that the job processing times are subject to changes due to the phenomenon of learning and/or controllable processing time (resource allocation). Job learning appears, for instance, in repeated processing of similar tasks which improves worker's skills; workers are able to perform setup, to handle raw materials and components at a greater pace, or to deal with machine operations and software (Biskup, 1999). Allocate a common limited resource such as financial budget, overtime, energy, fuel, subcontracting or manpower controls job-processing times.

Biskup (1999) and Cheng and Wang (2000) are pioneers who introduced Wright (1936) learning curve into the field of scheduling. Biskup (1999) assumed that the processing time of a job is a log-linear learning curve, i.e., if job J_j is scheduled in position r in a sequence, its actual processing time is

$$p_j = \bar{p}_j r^a$$

where $\bar{p}_j$ is the basic (normal) processing time of job J_j, $a \leqslant 0$ is a constant learning effect. He proved the polynomial solvability of single machine scheduling problems of minimizing the sum of job flow times and the total deviations of job completion times from a common due date. After him, a number of studies have been reported regarding the scheduling problem with learning effect in a single machine such as Kuo and Yang (2006), Cheng, Wu, and Lee (2008), Wu and Lee (2008), Lee, Wu, and Liu (2009), Janiak, Janiak, Rudek, and Wielgus (2009), Cheng, Lai, Wu, and Lee (2009), Zhu, Sun, Sun, and Li (2010), Cheng, Lee, and Wu (2010), Cheng, Wu, Cheng, and Wu (2011), Yin, Wu, Wu, and Cheng (2012), Yin, Cheng, and Wu (2012), Yin, Liu, Hao, and Zhou (2012). In the meanwhile, increasing efforts have been invested on scheduling with learning considerations in the flowshop environment. Lee and Wu (2004) minimized total completion time in a two-machine flowshop with a learning effect. Koulamas and Kyparisis (2007) demonstrated the solvability of total completion time minimization problems for the two-machine flowshop under the shortest processing time sequencing rule when the job processing times are ordered and job-position-based learning is in effect. Wu and Lee (2009) minimized the sum of completion times or flow times under a permutation flowshop scheduling problem with learning effects. Wang and Liu (2009) investigated a two-machine flowshop scheduling problem with both deterioration and learning effects. Yin, Xu, Sun, and Li (2009) utilized a model, in which, the

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actual processing time of a job position is a function of both total normal processing times and job's scheduling to show that some single machine scheduling problems and m-machine permutation flowshop problems are polynomially solvable. Li, Hsu, Wu, and Cheng (2011) considered a two-machine flowshop scheduling problem in which the actual processing time of a job is determined by the job's position and a control parameter of the learning function. Wang, Wu, and Ji (2012) proposed a revised model of the general learning effect and showed that m-machine flowshop scheduling problems are polynomially solvable under the revised model. Cheng, Wu, Chen, Wu, and Cheng (2013) studied a two-machine flowshop scheduling problem with a truncated learning function in which the processing time is a function of the job's position in a scheduling and the learning truncation parameter. They proposed a branch-and-bound and three crossover-based GA to find the optimal and approximate solutions of the minimum makespan, respectively. Under the job position-based learning model proposed by Biskup (1999), however, the actual processing time of a job drops to zero precipitously as the number of jobs increases, which is unrealistic. Motivated by this observation, Cheng, Cheng, Wu, Hsu, and Wu (2011) considered a two-agent single-machine scheduling problem with truncated sum-of-processing-times-based learning considerations. Wu, Wu, Hsu, and Lai (2012) developed a branch-and-bound and a genetic heuristic-based algorithm for a two-machine total completion time flowshop problem, in which the actual job processing time is a function depending on the jobs that have already been processed and a control parameter.

Meanwhile, sequencing problems with controllable processing times have been reported extensively by researchers since 1980. A survey of results on this subject, up to 1990, can be found in Nowicki and Zdrzałka (1990). A more recent literature review was given by Shabtay and Steiner (2007). In physical or economic systems, many resource allocation problems do not use a linear resource consumption function, since it fails to reflect the law of diminishing marginal returns. In order to account for this law, a number of studies on scheduling with resource allocation assumed that the job processing time is a convex decreasing function of the amount of resource allocated to the processing of the job (Shabtay (2004), Shabtay and Kaspi (2004)). Shabtay, Kaspi, and Steiner (2007) considered a no-wait two-machine flowshop scheduling problem with convex resource-dependent processing times at the first time, and proved this problem is strongly NP-hard. However, considering both the learning effects and the controllable processing times in a scheduling problem simultaneously, especially in two-machine flowshop scheduling problem has not been studied yet.

For a convex resource dependent function, we assume the following relationship between the job processing time and the resource allocated to the job:

$$p_j = \left(\frac{\bar{p}_j}{u_j}\right)^k, \quad u_j > 0,$$

where $\bar{p}_j$ is the non-compressed (normal) processing time of job J_j, u_j is the amount of resource allocated to job J_j and k is a positive constant. Since scheduling with controllable processing times is essentially a bacterial problem, which involves costs due to both a scheduling criterion and the total resource consumption, four different types of optimization models are applicable. The first one is to minimize a cost function that is the sum of the two costs (e.g., Vickson (1980), Alidaee and Ahmadian (1993), and Ng, Cheng, Kovalyov, and Lam (2003)). The second one is to minimize a scheduling criterion under a constraint on the total resource consumption (e.g., Janiak (1987) and Janiak and Szkodny (1994)). The third type is to minimize the total resource consumption with limited maximal value of a scheduling criterion (e.g., Cheng and Kovalyov (1995) and Janiak and Kovalyov (1996)) and the last one is to construct the trade-off curve between the scheduling criterion and resource consumption (e.g., Van Wassenhove and Baker (1982), Daniels and Sarin (1989) and Cheng, Janiak, and Kovalyov (1998)). In this study, we adopt the fourth approach.

The effects of learning and resource allocation have never been considered concurrently since Wang, Wang, and Wang (2010). They considered a single-machine scheduling problem with position-based learning effect and linear/convex resource-dependent processing times to minimize the weighted sum of the makespan of all jobs, the total completion/waiting times, the total absolute differences in completion/waiting times, and resource cost. Here, we study two-machine no-wait flowshop scheduling problem with position and resource-dependent processing times at the same time.

The rest of this paper is organized as follows. A formal description of the model under this study will be given in Section 2. Sections 3 and 4 will demonstrate the polynomial solvability of the proposed problems. In Section 5, a test example is given. Conclusions and discussion will be presented in Section 6.

2. Problem formulation

In order to simultaneously take into account the learning effects and resource allocation in a no-wait two-machine flowshop, a new model is proposed as follows:

There are a set $J = \{J_1, J_2, \ldots, J_n\}$ of n jobs scheduling on a no-wait two-machine flowshop. With the assumption that all the jobs are independent, non-preemptive and immediately available for processing. Each job J_j includes a set of two operations, where O_{ji} represents the operation of job J_j on machine i for $i = 1, 2$ and $j = 1, 2, \ldots, n$. Each job has to be processed on both machines following the same route and each machine handles one job at a time. We assume that jobs are not allowed to wait between the two machines, i.e., there is a no-wait restriction. Let p_j be the actual processing time of job J_j and p_{ji} be the actual processing time of operation O_{ji}. Here, we consider the following time and convex resource-dependent processing times model

$$p_{ji} = \left(\frac{\bar{p}_{ji} r^a}{u_{ji}}\right)^k, \quad u_{ji} > 0, \tag{1}$$

where $\bar{p}_{ji}$ is the basic(normal) processing time of operations O_{ji}, r is the position of job J_j which is scheduled in a sequence, $a \leqslant 0$ is a learning index, and u_{ji} is the amount of a non-renewable resource allocated to operations O_{ji} and k is a positive constant.

For a given sequence $\pi = [J_{[1]}, J_{[2]}, \ldots, J_{[n]}]$, the makespan in a no-wait two-machine flowshop problem can be represented as the longest path in a directed acyclic graph as shown in Fig. 1, where $[j]$ is the job in the jth position in π, with constraints $p_{[j]2} = p_{[j+1]1}$.

$C_j = C_j(\pi)$ represents the completion time for job J_j. Let $C_{\max} = \max\{C_j | j = 1, 2, \ldots, n\}$, $TC = \sum_{j=1}^n C_j, TW = \sum_{j=1}^n W_j, TADC = \sum_{h=1}^n \sum_{j=h}^n |C_h - C_j|$, and $TADW = \sum_{h=1}^n \sum_{j=h}^n |W_h - W_j|$ be the makespan of all jobs, the total completion times, the total waiting times, the total absolute differences in completion times, and the total absolute differences in waiting times, respectively, where $W_j = C_j - p_j$ is the waiting time of job J_j. The objective is to determine the optimal resource allocations and the optimal sequence of jobs in the flowshop so that the corresponding value of the following cost functions be optimal:

$$f_1(\pi, u) = \delta_1 C_{\max} + \delta_2 TC + \delta_3 TADC + \delta_4 \sum_{j=1}^n G_j u_j, \tag{2}$$

$$f_2(\pi, u) = \delta_1 C_{\max} + \delta_2 TW + \delta_3 TADW + \delta_4 \sum_{j=1}^n G_j u_j, \tag{3}$$