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Technical Note

Notes on "Possibilistic programming approach for fuzzy multidimensional analysis of preference in group decision making" ☆

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ABSTRACT

The aim of this note is to point out and correct some errors in the definitions, notations operations and possibilistic programming model introduced by Sadi-Nezhad and Akhtari (2008) and hereby develop two correct possibilistic programming models for fuzzy multidimensional analysis of preference in the fuzzy multiattribute group decision making problems with both the fuzzy weight vector and the fuzzy positive ideal solution (PIS) unknown *a priori*.

1. Some errors in Sadi-Nezhad and Akhtari's paper and analysis

Sadi-Nezhad and Akhtari (2008) studied the following fuzzy multiattribute group decision making (FMAGDM) problem: the group of P decision makers P_p $(p = 1, 2, \ldots, P)$ has to choose one of or rank m alternatives A_i $(i = 1, 2, \ldots, m)$ based on n attributes C_j $(j = 1, 2, \ldots, n)$. Denote the alternative set by $A = \{A_1, A_2, \ldots, A_m\}$ and the attribute set by $C = \{C_1, C_2, \ldots, C_n\}$. Let $\tilde{x}_{ij}$ be the fuzzy score of an alternative A_i $(i = 1, 2, \ldots, m)$ on each attribute C_j $(j = 1, 2, \ldots, n)$ and $\widetilde{W}_j$ be the fuzzy weight of an attribute C_j, where $\tilde{x}_{ij}$ and $\widetilde{W}_j$ are triangular fuzzy numbers (Dubois & Prade, 1980), denoted by $\tilde{x}_{ij} = (a_{ijL}, a_{ijM}, a_{ijR})$ and $\widetilde{W}_j = (w_{jL}, w_{jM}, w_{jR})$, respectively. Here, we stipulate: $a_{ijL} \leqslant a_{ijM} \leqslant a_{ijR}$ and $0 \leqslant w_{jL} \leqslant w_{jM} \leqslant w_{jR}$. Thus, the above FMAGDM problem can be concisely expressed in the matrix format as follows:

$$\tilde{D} = (\tilde{x}_{ij})_{m\times n} = \begin{matrix} & C_1 & C_2 & \cdots & C_n \\ A_1 & \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ A_2 & \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ A_m & \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{matrix} \quad (1)$$

Assume that the decision makers P_p $(p = 1, 2, \ldots, P)$ express the preference relations between alternatives with the fuzzy sets of ordered pairs of the alternatives, denoted by $\tilde{\Omega}_p = \{((k,l), \widetilde{C}_p(k,l)) | k = 1, 2, \cdots, m; l = 1, 2, \cdots, m\}$, where (k,l) expresses an ordered pair of the alternatives A_k and A_l that the decision maker P_p prefers A_k to A_l with the degree of truth $\widetilde{C}_p(k,l)$, and $\widetilde{C}_p(k,l)$ is a triangular fuzzy number defined on the unit interval [0,1], denoted by $\widetilde{C}_p(k,l) = (C^p_{klL}, C^p_{klM}, C^p_{klR})$, which satisfies the condition: $0 \leqslant C^p_{klL} \leqslant C^p_{klM} \leqslant C^p_{klR} \leqslant 1$.

According to the idea of the fuzzy LINMAP method (Li & Yang, 2004; Srinivasan & Shocker, 1973), Sadi-Nezhad and Akhtari (2008) constructed the possibilistic programming model for the above FMAGDM problem with the fuzzy weight vector $\widetilde{W} = (\widetilde{W}_1, \widetilde{W}_2, \cdots, \widetilde{W}_n)$ and the fuzzy positive ideal solution (PIS) $\tilde{a}^* = (\tilde{a}^*_1, \tilde{a}^*_2, \cdots, \tilde{a}^*_n)$ unknown *a prior*, where $\widetilde{W}_j$ and $\tilde{a}^*_j = (a^*_{jL}, a^*_{jM}, a^*_{jR})$ $(j = 1, 2, \ldots, n)$ are triangular fuzzy numbers. However, it is found that there are the following errors in the definitions, notations and operations and possibilistic programming model introduced by Sadi-Nezhad and Akhtari (2008).

(A) The decision variables of the objective functions in Eqs. (9) and (11) of Sadi-Nezhad and Akhtari (2008) are incorrectly expressed as λ^p_{kl} instead of λ_{kl}. In fact, according to Eq. (1) in this note (i.e., Eq. (7) of Sadi-Nezhad and Akhtari (2008), the fuzzy distance between an alternative A_i and the fuzzy PIS $\tilde{a}^*$ is defined as follows (Sadi-Nezhad and Akhtari, 2008):

$$\widetilde{S}_i = \sum_{j=1}^{n} \widetilde{W}_j (\tilde{x}_{ij} - \tilde{a}^*_j)^2 \quad (2)$$

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i.e., Eq. (8) of Sadi-Nezhad and Akhtari (2008). Obviously, $\widetilde{S}_i$ is not related to the decision maker P_p. In other words, $\widetilde{S}_i$ is independent of the subscript of the decision maker P_p. Hence, $\max\{0, \widetilde{S}_l - \widetilde{S}_k\}$ is independent of the decision maker P_p, i.e., $\max\{0, \widetilde{S}_l - \widetilde{S}_k\}$ should be rightly denoted by λ_{kl} instead of λ_{kl}^p.

(B) The decision variables λ_{kl}^p in Eqs. (9) and (11) of Sadi-Nezhad and Akhtari (2008) (i.e., the correct notation λ_{kl} in this note) are incorrectly regarded as non-negative real numbers instead of non-negative triangular fuzzy numbers. In fact, according to the operations of triangular fuzzy numbers (Dubois and Prade, 1980), $\widetilde{S}_i$ is a triangular fuzzy number since all $\widetilde{W}_j$, $\tilde{x}_{ij}$ and $\tilde{a}_j^*$ (j = 1, 2, ..., n) in Eq. (2) are triangular fuzzy numbers. Thus, $\widetilde{S}_l - \widetilde{S}_k$ is a triangular fuzzy number. Hence, all the incorrect notations $\lambda_{kl}^p = \max\{0, \widetilde{S}_l - \widetilde{S}_k\}$ of Sadi-Nezhad and Akhtari (2008) (i.e., the correct notations λ_{kl} of this note) for $(k,l) \in \Omega^p$ should be non-negative triangular fuzzy numbers instead of non-negative real numbers.

(C) The threshold h in the constraints of Eqs. (9) and (11) of Sadi-Nezhad and Akhtari (2008) is incorrectly assumed to be a positive constant (i.e., a real number) instead of a positive triangular fuzzy number. Such an incorrect hypothesis results in the right-hand side of the corresponding equality in the constraint of Eq. (9) of Sadi-Nezhad and Akhtari (2008) must be a positive real number and is equal to h, i.e., $\sum_{p=1}^{P}\sum_{(k,l)\in\tilde{\Omega}_p}(\widetilde{S}_l - \widetilde{S}_k)$ must be a real number and is equal to h. However, according to the operations of triangular fuzzy numbers (Dubois & Prade, 1980) and similar analysis in the above case (B) of this note, $\sum_{p=1}^{P}\sum_{(k,l)\in\tilde{\Omega}_p}(\widetilde{S}_l - \widetilde{S}_k)$ should be a triangular fuzzy number instead of a real number since both $\widetilde{S}_k$ and $\widetilde{S}_l$ are triangular fuzzy numbers. Analogously, it is found that there is a similar error in the constraint of Eq. (11) of Sadi-Nezhad and Akhtari (2008).

(D) All the inequalities $\widetilde{S}_k - \widetilde{S}_l + \lambda_{kl}^p \geqslant 0$ for $(k,l) \in \Omega^p$ ($p = 1, 2, \ldots, P$) in the constraints of Eq. (9) of Sadi-Nezhad and Akhtari (2008) are not right. In fact, these inequalities should be $\widetilde{S}_l - \widetilde{S}_k + \lambda_{kl}^p \geqslant 0$ for $(k,l) \in \Omega^p$ ($p = 1, 2, \ldots, P$) since $\lambda_{kl}^p = \max\{0, \widetilde{S}_k - \widetilde{S}_l\}$ (i.e., the correct notation λ_{kl} in this note) is the inconsistency index between the ranking order of the alternatives A_k and A_l determined by $\widetilde{S}_l$ and $\widetilde{S}_k$ and the preference of the decision maker P_p preferring A_k to A_l. Otherwise, we obtain $\lambda_{kl}^p = \max\{0, \widetilde{S}_l - \widetilde{S}_k\}$ which is the consistency index between the ranking order of the alternatives A_k and A_l determined by $\widetilde{S}_l$ and $\widetilde{S}_k$ and the preference of the decision maker P_p preferring A_k to A_l. In this case, min $\sum_{p=1}^{P}\sum_{(k,l)\in\tilde{\Omega}_p}\widetilde{C}_p(k,l)$ $\lambda_{kl}^p\}$ of Eq. (9) of Sadi-Nezhad and Akhtari (2008) means minimizing the total consistency index of the group, which is not rational.

(E) There are the following errors appearing in the process of Eq. (9) being transformed into Eq. (11) in Sadi-Nezhad and Akhtari (2008).

(E1) The coefficients of the objective functions Z_1, Z_2 and Z_3 in Eq. (11) of Sadi-Nezhad and Akhtari (2008) are incorrectly expressed as $C_{klM} - C_{klL}$, C_{klM} and $C_{klR} - C_{klM}$ instead of $C_{klM}^p - C_{klL}^p$, C_{klM}^p and $C_{klR}^p - C_{klM}^p$, respectively.

(E2) The three conditions $\sum_{j=1}^{m} w_{jL} \leqslant 1$, $\sum_{j=1}^{m} w_{jM} \leqslant 1$ and $\sum_{j=1}^{m} w_{jR} = 1$ are incorrectly imposed on the constraints of Eq. (11) of Sadi-Nezhad and Akhtari (2008), which may result in greatly minishing the range of feasible solutions of the fuzzy weight vector $\widetilde{W}$.

(E3) All the inequalities $\upsilon_{jL} \leqslant \upsilon_{jM}$ and $\upsilon_{jM} \leqslant \upsilon_{jR}$ ($j = 1, 2, \ldots, n$) are incorrectly imposed on the constraints of Eq. (11) of Sadi-Nezhad and Akhtari (2008).

(E4) All the inequalities $\sum_{j=1}^{m} w_{jL}(a_{ljL}^2 - a_{kjL}^2) - 2\sum_{j=1}^{m}\upsilon_{jL}$ $(a_{ljL} - a_{kjL}) + \lambda_{kl}^p \geqslant 0$, $\sum_{j=1}^{m} w_{jM}(a_{ljM}^2 - a_{kjM}^2) - 2\sum_{j=1}^{m}\upsilon_{jM}(a_{ljM} - a_{kjM}) + \lambda_{kl}^p \geqslant 0$ and $\sum_{j=1}^{m} w_{jR}(a_{ljR}^2 - a_{kjR}^2) - 2\sum_{j=1}^{m}\upsilon_{jR}(a_{ljR} - a_{kjR}) + \lambda_{kl}^p \geqslant 0$ for $(k,l) \in \tilde{\Omega}_p$ ($p = 1, 2, \ldots, P$) in Sadi-Nezhad and Akhtari (2008) are not right. In fact, according to Eqs. (8) and (9) of Sadi-Nezhad and Akhtari (2008) and the analysis in the above case (A), these inequalities should be correctly written as $\sum_{j=1}^{m} w_{jL}(a_{kjL}^2 - a_{ljL}^2) - 2\sum_{j=1}^{m}\upsilon_{jL}(a_{kjL} - a_{ljL}) + \lambda_{kl} \geqslant 0$, $\sum_{j=1}^{m} w_{jM}(a_{kjM}^2 - a_{ljM}^2) - 2\sum_{j=1}^{m}\upsilon_{jM}$ $(a_{kjM} - a_{ljM}) + \lambda_{kl} \geqslant 0$ and $\sum_{j=1}^{m} w_{jR}(a_{kjR}^2 - a_{ljR}^2) - 2\sum_{j=1}^{m}\upsilon_{jR}(a_{kjR} - a_{ljR}) + \lambda_{kl} \geqslant 0$ for $(k,l) \in \tilde{\Omega}_p$ ($p = 1, 2, \ldots, P$), respectively. However, it is obvious that these inequalities are independent of the decision makers P_p. Thus, the FMAGDM problem discussed in Sadi-Nezhad and Akhtari (2008) is not proper "group" decision making. In other words, Sadi-Nezhad and Akhtari (2008) essentially discussed the fuzzy multiattribute decision making problem with only one decision maker rather than two or more than two decision makers. Namely, Sadi-Nezhad and Akhtari (2008) incorrectly used the term "group decision making".

(E5) The equalities in Eq. (12) of Sadi-Nezhad and Akhtari (2008) are not right. They should be $\upsilon_{jL} = w_{jL}a_{jR}^*$, $\upsilon_{jM} = w_{jM}a_{jM}^*$ and $\upsilon_{jR} = w_{jR}a_{jL}^*$ ($j = 1, 2, \ldots, n$), respectively.

(F) The linear programming model in Appendix A of Sadi-Nezhad and Akhtari (2008) is inconsistent with both Eqs. (9) and (11) since the inequalities $W_{jL} \geqslant 0.001$ ($j = 1, 2$) are incorrectly imposed on the constraints of the linear programming model constructed by Sadi-Nezhad and Akhtari.

(G) The computation results of the two examples of Sadi-Nezhad and Akhtari (2008) are not right. For example, the solution given by $\widetilde{W}_1 = (0.0010, 0.0095, 0.202)$, $\widetilde{W}_2 = (0.0012, 0.0012, 0.0012)$, $\widetilde{V}_1 = (0.0024, 0.0063, 0.0152)$ and $\widetilde{V}_2 = (0.0027, 0.0027, 0.0050)$ is not feasible to the linear programming model of the hypothetical study (Section 4.1) in Appendix A of Sadi-Nezhad and Akhtari (2008) since

$$\begin{aligned}&-0.91W_{1M} - 1.91W_{2M} + 1.0V_{1M} + 3.8V_{2M}\\ &= -0.91 \times 0.0095 - 1.91 \times 0.0012 + 1.0 \times 0.0063 + 3.8 \times 0.0027\\ &= 0.0056 \neq 0.01\end{aligned}$$

and

$$\begin{aligned}&-1.26W_{1R} - 3.23W_{2R} + 1.2V_{1R} + 4.2V_{2R}\\ &= -1.26 \times 0.202 - 3.23 \times 0.0012 + 1.2 \times 0.0152 + 4.2 \times 0.0050\\ &= -0.219 \neq 0.01,\end{aligned}$$

i.e., these two equalities of the constraints in the linear programming model are not valid.

2. Correctly developed possibilistic programming models for fuzzy multiattribute group decision making

Stated as earlier, the possibilistic programming model and method proposed by Sadi-Nezhad and Akhtari (2008) could be applicable only if the errors in the above definitions, notations, operations, and model were corrected. As a result, in the sequent, we give the correct definitions, notations, operations and hereby correctly propose two possibilistic programming models for the above FMAGDM problems.

In the FMAGDM problem, the decision matrixes of the decision makers P_p ($p = 1, 2, \ldots, P$) are correctly expressed as follows:

$$\tilde{D}^p = (\tilde{x}_{ij}^p)_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array}\!\!\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{pmatrix} \tilde{x}_{11}^p & \tilde{x}_{12}^p & \cdots & \tilde{x}_{1n}^p \\ \tilde{x}_{21}^p & \tilde{x}_{22}^p & \cdots & \tilde{x}_{2n}^p \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{m1}^p & \tilde{x}_{m2}^p & \cdots & \tilde{x}_{mn}^p \end{pmatrix} \end{array}, \quad (3)$$