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Performance of t control charts in short runs with unknown shift sizes ☆

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ABSTRACT

Recently, control charts plotting a statistic having a Student's t distribution have been proposed as an efficient solution to perform Statistical Process Control (SPC) in short production runs where the shift size of the in-control process mean from μ_0 to μ_1 is known *a priori*. The shift size is usually measured as a multiple δ of the in-control process standard deviation σ_0: but in practice, at the beginning of the production run, both the value of next shift δ and σ_0 are *unknown*. As a consequence, when the actual shift size differs from the value assumed at the chart design stage, the performance of the control chart can be seriously affected. To overcome this problem, this paper investigates the statistical performance of the Shewhart, EWMA and CUSUM t charts for short production runs when the shift size is *unknown* and modeled by means of a statistical distribution. An extensive numerical analysis allows the properties of the three charts to be compared and discussed when uniform and triangular distributions are used by quality practitioners to fit the unknown shift size. An illustrative example is utilized to demonstrate a practical implementation of the best performing among the three investigated charts.

1. Introduction

Among quality practitioners it is common practice to perform online quality control by means of control charts. A control charting procedure consists of getting a sample of n measures of a quality characteristic of a product at fixed time intervals and plotting on a graph a statistic computed through the n measures vs. a control interval delimited by two control limits having width proportional to a parameter k, Montgomery (2008). If a point is plotted outside the control interval, then a potential process shift to the *out-of-control* condition is triggered by the chart and the search procedures for an assignable cause or a false alarm should be started. Control charts for long run processes were originally proposed by W.A. Shewhart while working for Bell Laboratories in 1920s. Later, CUSUM and EWMA control charts were respectively proposed by Page (1954) and Roberts (1959) to improve the detection capabilities with respect to small to moderate process shifts to the out-of-control condition.

Implementing control charts in short production runs allowing for a limited number of scheduled inspections is a challenging issue, which currently receives attention in quality control literature. In fact, the current flexibility and modularity of manufacturing systems, along with a more efficient management of operations, allows an increasingly large number of companies to embrace the Just-in-Time (JIT) philosophy and then to switch frequently between similar codes with actually short set-up times. As an example, short production runs of few hundreds of identical items are common in job shop manufacturing of small series of mechanical parts by means of multi-purpose Computer Numerical Control (CNC) machines and in manufacturing processes of customized products like it usually happens in the clothes, fashion and furniture industry. Furthermore, there are high volume manufacturing processes where the rolling horizon scheduled for the production of a product code is such that the number of allowed inspections during the production run can be limited by the rate of inspection of the quality characteristic to be monitored. For example, in several beverage industries the switch from filling with soft drink one bottle size to another occurs after a batch consisting of several thousands of bottles, according to the production planning decisions. Usually, a reconfiguration process set-up is performed on the basis of a shift work pattern every 24–32 h, thus limiting the production horizon for each bottle size. In the soft drink manufacturing processes, the measurement of the percentage of carbonation obtained by adding carbon dioxide to the soft drink is an important quality issue, which strongly depends on the material and the size of the bottle to be filled. Thus, SPC should be started and performed sequentially on small samples of bottles within each batch production run. However, although the production rate

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of the soft drink manufacturing processes is high, the measurement test used to get the correct value of percentage of carbonation inside each bottle requires several minutes to be completed: for this reason, a limited number of bottles of the same size can be inspected every eight hours, thus limiting to 20–30 the number of scheduled inspections between two successive set-ups.

In such process operating conditions, data about the current in-control population mean μ_0 and standard deviation σ_0 of the quality characteristic are usually not available before production starts and the quality practitioner cannot get enough sample statistics by means of the implementation of a *Phase I* control chart because of the limited horizon of production and the small number of inspections. Furthermore, information about the size δ of the mean shift is unavailable before the start of a run and the chart can have a poor performance if the actual size occurring during the run is different than the size value assumed at the chart design stage. This lack of preliminary information would lead quality practitioners to exclude the implementation of a control chart as an on-line monitoring tool during a short production run with finite number of inspections.

To overcome this problem, in this paper we propose and investigate an approach to get the statistical design of the Shewhart, EWMA and CUSUM *t* charts for short production runs when the shift size of the process mean is *unknown*. The remainder of the paper is organized as follows: in the next Section a literature review is presented; then, in Section 3, the three control charts are briefly commented. In Section 4, the statistical measures adopted to compare the charts are presented. Section 5 presents the discussion regarding the performance comparisons. In Section 6, an example is provided to illustrate a practical implementation of the best performing chart among the three investigated *t* control charts. Conclusions and an Appendix briefly presenting the stochastic model adopted to compute the statistical properties of the three charts complete the paper.

2. Literature review

When historical data about μ_0 and σ_0 of the quality characteristic are usually not available before a production starts, literature about control charts has investigated the self-starting control chart schemes based on parameters estimates, which are updated with each new observation, see Hawkins (1987), Quesenberry (1991, 1995), Tsai, Lin, and Wu (2004, 2005), Hawkins and Maboudou-Tchao (2007), He, Jiang, and Shu (2008), Li, Zhang, and Wang (2009). However, these self-starting control charts require a minimum number of samples to get good parameters estimates and their statistical properties are generally computed with respect to the long run context. In particular, Quesenberry (1991, 1995) has defined a set of sequential *Q* statistics to be used in Shewhart, EWMA and CUSUM charts for the detection of changes in the process mean or variance. He et al. (2008) have investigated the statistical properties of the *Q* charts and have demonstrated that they can be biased: that is, if the mean shifts in the early stage of charting, then the *Q* statistic is affected by this shift and the control chart has a larger out-of-control average run length (*ARL*) than its in-control *ARL*. This problem is known as the "masking of the shift" and can be *critical* if the shift occurs at the early stage of production, which can be frequent in manufacturing processes where the number of scheduled inspections are limited to a few. The modified *Q* control charts proposed by He et al. (2008) still present some bias and difficulty of implementation in practice. A complex control charting scheme which updates the reference pattern of a CUSCORE-type chart using an adaptive EWMA has been recently proposed by Capizzi and Masarotto (2012). More recently, Li and Pu (2012) have investigated the statistical properties of control charts for short runs by comparing several performance metrics in presence of a fixed shift size.

Another stream of research considers Bayesian control charts for short runs, where the sample size, sampling interval and width of control limits are continuously updated by using the Bayes' theorem, see Tagaras (1996) and Nenes and Tagaras (2007): these Bayesian control charting schemes assume as ***known a priori*** the in-control parameters μ_0 and σ_0 and the shift size δ.

Recently, *t* control charts have been proposed as an efficient quality control tool in a short production run, Celano, Castagliola, Trovato, and Fichera (2011a). These charts monitor a statistic having a Student's *t* distribution and are managed and interpreted exactly as the traditional charts with known parameters. It is worth pointing out that these charts have nothing in common with the *t* chart used to monitor the time interval (*TBE*) between the occurrences of an event: for further details about these other charts, see Xie, Goh, and Ranjan (2002), and Wu, Jiao, and He (2009). For a *long run process*, Zhang, Chen, and Castagliola (2009) compared the statistical properties of Shewhart and EWMA *t* and $\overline{X}$ charts when the in-control process standard deviation is not correctly estimated. Gao (2011) has recently shown that the *t* chart for short production runs can be an efficient SPC monitoring tool in mixed model assembly production. The implementation of Shewhart and EWMA *t* control charts to monitor the process mean in a *short production run* with a finite number of inspections has been firstly investigated by Celano et al. (2011a): the main motivation leading to the *t* charts implementation in short production runs is that any preliminary process parameter estimation is unnecessary and SPC can be started *immediately* with the production run. In Celano et al. (2011a), the statistical properties of the Shewhart and EWMA *t* charts are compared by assuming the shift size δ of the process mean from μ_0 to $\mu_1 = \mu_0 + \delta\sigma_0$ as *fixed and known a priori*. The economic design of the Shewhart *t* chart with fixed shift size has been recently investigated by Celano, Castagliola, Fichera, and Trovato (2012).

In practice, the next shift size is never known and cannot be exactly predicted *a priori*. A recent review about the effect of random shift size on the economic design of different chart schemes implemented in long run processes is available in Celano (2011). The shift size can get different values during a production run, as consecutive assignable causes may occur. It is well known that a control chart has a poor statistical performance when its design is selected to be optimal for an assumed fixed value δ of the shift size and, then, its actual value occurring during production is different from that selected at the design stage. For this reason, the problem of unknown shift size is receiving careful attention in SPC literature. Two approaches have been proposed *only* for production processes having *infinite* rolling horizon:

- The *non-parametric* approach, see Wu, Shamsuzzaman, and Pan (2004), calls for a preliminary collection of a historical set of shift sizes d_i during the process run. Then, quality practitioners get the design of the chart by optimizing a weighted average of the control chart's statistical/economic performance computed by starting from this historical dataset of shift sizes.
- The *parametric* approach fits a theoretical statistical distribution to the collected values d_i. In this case, an expected value of the selected measure of the control chart's performance is computed vs. the fitting statistical distribution.

Several statistical distributions have been considered to implement the parametric approach. The Rayleigh probability distribution has been introduced by Wu et al. (2004, 2006) and Jiao and Helo (2008) to fit the true distribution of a unimodal random shift size. The normal distribution has been used by Chen and Chen (2007) and Ryu, Wan, and Kim (2010) to model the unknown shift size and to get the design for EWMA and CUSUM charts monitoring the sample mean. It is worth noting that both the Rayleigh and the