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Monitoring general linear profiles using simultaneous confidence sets schemes ☆

Longcheen Huwang [a,*], Yi-Hua Tina Wang [b], Shuhan Xue [a], Changliang Zou [c]

[a] Institute of Statistics, National Tsing Hua University, Hsinchu, Taiwan
[b] Department of Statistics, Tamkang University, Tamsui, Taiwan
[c] LPMC and Department of Statistics, Nankai University, Tianjin, China

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ABSTRACT

In this article we consider the quality of a process which can be characterized by a general linear profile. For monitoring the general linear profile, we mimic the charting scheme for the distribution of a univariate quality characteristic by using two individual charts for the mean and variance of the profile, respectively. For monitoring the mean of the profile, based on the concept of simultaneous confidence set we propose a novel exponentially weighted moving average (EWMA) chart, which takes the features of the entire profile into account. Then this chart is used together with an EWMA chart for the variance of the profile to monitor the whole profile. Simulation studies show the effectiveness and efficiency of the proposed monitoring scheme. Furthermore, a systematic diagnostic method in the literature is utilized to find the change point location and to identify the parameters of change in the process. Finally, we use an example from semiconductor manufacturing industry to demonstrate the implementation of the proposed monitoring scheme and diagnostic method.

1. Introduction

Statistical process control (SPC) has been successfully applied to monitor various industrial processes. In most SPC applications, the quality of a process can be adequately represented by the distribution of a univariate quality characteristic or by the multivariate distribution of a vector consisting of a few quality characteristics. In many applications, however, the quality of a process or product is characterized and summarized better by a relationship (or profile) between the response variable and one or more explanatory variables; that is, the main topic is on monitoring the profile that represents such a relationship, instead of on monitoring a single quality characteristic or several quality characteristics.

For monitoring simple linear profiles, Kang and Albin (2000) proposed two different control charting schemes in Phase I and Phase II monitoring. One of them is a multivariate T^2 chart and the other is the combination of an EWMA chart and a range (R) chart. Kim, Mahmoud, and Woodall (2003) proposed using a combination of three EWMA charts to respectively detect shifts in the intercept, slop, and standard deviation simultaneously in the Phase II monitoring. They also proposed applying similar Shewhart-type control charts in the Phase I monitoring. Gupta, Montgomery, and Woodall (2006) compared the performance of the control charts proposed by Croarkin and Varner (1982) and Kim et al. (2003) for monitoring simple linear profiles in the Phase II study. They concluded that Kim et al.'s combined EWMA charts are better than Croarkin and Varner's charting scheme. Mahmoud and Woodall (2004) studied several control charting schemes for monitoring simple linear profiles in the Phase I study. Zou, Zhang, and Wang (2006) proposed a control charting scheme on the basis of a change point model for monitoring simple linear profiles where the process parameters are unknown but can be estimated from the in-control historical data. Mahmoud, Parker, Woodall, and Hawkins (2007), based on likelihood ratio statistics, proposed a change point method for detecting sustained shifts in simple linear profiles in the Phase I study. Zou, Zhou, Wang, and Tsung (2007a) studied a self-starting control chart for monitoring simple linear profiles when the process parameters are unknown but some in-control data in the Phase I study are available. For monitoring general linear profiles, Zou, Tsung, and Wang (2007b) applied an MEWMA single chart to the transformations of estimated profile parameters in the Phase II study. More studies for monitoring linear profiles can be found in the literature. See, e.g., Jensen, Hui, and Ghare (1984), Mestek, Pavlik, and Suchanek (1994), Stover and Brill (1998), Lawless, Mackay, and Robinson (1999).

Although monitoring linear profiles is an important issue, in many practical applications the profiles cannot be represented by linear models adequately. Walker and Wright (2002) studied

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* Corresponding author. Tel.: +886 3 5735385; fax: +886 3 5728318.
E-mail address: huwang@stat.nthu.edu.tw (L. Huwang).

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vertical density profiles which apparently cannot be represented by linear profiles. Woodall, Spitzner, Montgomery, and Gupta (2004) proposed control charts to monitor the same vertical density profiles. Williams, Woodall, and Birch (2007a) developed three general approaches to the formulation of T^2 statistics based on nonlinear model estimation in the Phase I study. Colosimo and Pacella (2007) employed principal component analysis to identify systematic patterns in roundness profiles. Williams, Birch, Woodall, and Ferry (2007b) utilized data from DuPont to monitor dose–response profiles used in high-throughput screening based on the nonlinear model approaches of Williams et al. (2007a), where a four-parameter logistic regression model was used to describe the profiles. Yeh, Huwang, and Li (2009) proposed Phase I profile monitoring schemes for binary responses that can be represented by the logistic regression model. Shang, Tsung, and Zou (2011) developed a control chart by integrating the EWMA scheme and the likelihood ratio test based on the logistic regression model in the Phase II study. Jin and Shi (1999) applied dimension-reduction techniques to study a stamping tonnage profile, which apparently is a nonlinear profile. Lada, Lu, and Wilson (2002) and Ding, Zeng, and Zhou (2006) used dimension-reduction techniques, including wavelet and independent component analysis to study a general category of nonlinear profiles.

Recently, Zou, Tsung, and Wang (2008) integrated an MEWMA procedure with a generalized likelihood ratio test (Fan, Zhang, & Zhang, 2001) based on the local linear smoother of Fan and Gijbels (1996) to develop a nonparametric control chart for monitoring general smooth regression profiles. Qiu, Zou, and Wang (2010) proposed monitoring smooth profiles which can be described by a nonparametric mixed-effects model to account for the within-profile correlation.

In this article we focus on the study of Phase II monitoring for a general linear profile. Based on the simultaneous confidence set's concept we propose a new EWMA chart, which takes account of the features of the entire profile, for monitoring the mean of the profile. Then this chart is combined with an EWMA chart for the variance of the profile to jointly monitor the whole profile. This approach originates from the concept of monitoring the univariate quality characteristic where two individual charts for the mean and variance are used jointly. It is different from all existing schemes for monitoring general linear profiles which monitor the whole profile through detecting changes in the parameters of the profile.

The rest of the paper is organized as follows. In Section 2 we review some existing schemes for monitoring general linear profiles. In Section 3, we present our monitoring approach in details for general linear profiles. Section 4 compares the performances of the proposed control charts and the existing ones. Section 5 provides a profile diagnosis method in the literature to estimate the change point and identify the parameters of change in the profile. In Section 6, an example from semiconductor manufacturing industry is used to illustrate the applicability of the proposed scheme. Conclusions and comments are given in the last section.

2. The existing monitoring schemes for general linear profiles

In this section, first we will briefly introduce the general linear profile that we are interested in monitoring. Then some popular monitoring schemes in the literature for the general linear profile will be reviewed.

Assume that for the jth sample collected overtime, $j = 1, 2, \ldots$, we have the observations $(X_j, \mathbf{y}_j)$, where $\mathbf{y}_j = (y_{j1}, y_{j2}, \ldots, y_{jn_j})'$ is an n_j-variate vector and X_j is a $n_j \times p$ $(n_j > p)$ matrix. Precisely, when the process is in control, the underlying model is assumed to be

$$\boldsymbol{y}_j = X_j\boldsymbol{\beta} + \boldsymbol{\varepsilon}_j, \quad (1)$$

where $\boldsymbol{\beta} = (\beta^{(1)}, \beta^{(2)}, \ldots, \beta^{(p)})'$ is a p-dimensional coefficient vector and $\boldsymbol{\varepsilon}_j = (\varepsilon_{j1}, \varepsilon_{j2}, \ldots, \varepsilon_{jn_j})'$ is a vector of n_j independent, identically distributed normal random variables with mean 0 and variance σ^2. Here we assume that X_j is of form $(\mathbf{1}, X_j^*)$, where each column of X_j^* is orthogonal to $\mathbf{1}$ and $\mathbf{1}$ is an n_j-variate vector of all 1's. Otherwise, we can obtain this form through some appropriate transformations. The explanatory variable matrix X_j is usually the same for different j and the n_j's are equal in practical applications (hereafter X and n are used to replace X_j and n_j).

The simple linear model, the simplest case of model (1), is the first profile used for describing the quality of a process. Kang and Albin (2000), Kim et al. (2003), Mahmoud and Woodall (2004), Mahmoud et al. (2007) proposed control charts for monitoring the simple linear profile. Assuming for the jth random sample collected overtime, we have the observations (x_i, y_{ij}), where y_{ij} is the response variable and x_i is the explanatory variable. When the process is in control, the underlying profile is as follows:

$$y_{ij} = A_0 + A_1 x_i + \varepsilon_{ij}, \quad i = 1, 2, \ldots, n, \quad (2)$$

where ε_{ij} are iid $N(0, 1)$ random variables. Kim et al. (2003) coded the explanatory variables to obtain the following model:

$$y_{ij} = B_0 + B_1 x_i^* + \varepsilon_{ij}, \quad i = 1, 2, \ldots, n, \quad (3)$$

where $B_0 = A_0 + A_1\bar{x}$, $B_1 = A_1$, $x_i^* = x_i - \bar{x}$, and $\bar{x} = \sum_{i=1}^n x_i/n$. For the jth profile, the least squares estimators for B_0, B_1, and σ^2 in model (3) are

$$B_{0j} = \bar{y}_j, \quad B_{1j} = \frac{S_{xy(j)}}{S_{xx}}, \quad \text{and } \hat{\sigma}_j^2 = \frac{1}{n-2}\sum_{i=1}^{n}\left(y_{ij} - B_{0j} - B_{1j}x_i^*\right)^2, \quad (4)$$

where $\bar{y}_j = \sum_{i=1}^n y_{ij}/n$, $S_{xx} = \sum_{i=1}^n (x_i - \bar{x})^2$, and $S_{xy(j)} = \sum_{i=1}^n (x_i - \bar{x})y_{ij}$. Note that the above three parameter estimators are mutually independent, and any control charting schemes based on these three estimators are regression invariant. For monitoring the simple linear profile (3), Kim et al. (2003) used the combination of three EWMA charts to jointly detect shifts in B_0, B_1, and σ^2 of the profile. Denote

$$\mathrm{EWMA}_I(j) = \lambda B_{0j} + (1-\lambda)\mathrm{EWMA}_I(j-1),$$
$$\mathrm{EWMA}_S(j) = \lambda B_{1j} + (1-\lambda)\mathrm{EWMA}_S(j-1),$$
$$\mathrm{EWMA}_{E^+}(j) = \max\{\lambda \ln(\hat{\sigma}_j^2/\sigma^2) + (1-\lambda)\mathrm{EWMA}_{E^+}(j-1), 0\},$$

and

$$\mathrm{EWMA}_{E^-}(j) = \min\{\lambda \ln(\hat{\sigma}_j^2/\sigma^2) + (1-\lambda)\mathrm{EWMA}_{E^-}(j-1), 0\},$$

where $\mathrm{EWMA}_I(0) = B_0$, $\mathrm{EWMA}_S(0) = B_1$, $\mathrm{EWMA}_{E^+}(0) = \mathrm{EWMA}_{E^-}(0) = 0$, and λ, $(0 < \lambda \le 1)$, is a smoothing constant. Note that the EWMA statistic defined above for monitoring σ^2 is slightly different from that of Kim et al. (2003) because the variance estimator $\hat{\sigma}_j^2$ has been scaled by σ^2 therein. Also, the EWMA statistic for detecting decrease shifts in σ^2 is provided as well. This is something that did not receive adequate attention in Kim et al. (2003), where only detecting increase shifts in σ^2 was considered. These four EWMA charts are used jointly, and the profile is considered to be out of control if at least one of the four charts triggers a signal. The four combined charts will be denoted as the KMW charts in this article.

For monitoring the general linear profile (1), Zou et al. (2007b) introduced a single MEWMA chart to detect changes in the $p+1$ parameters, the p coefficients $\beta^{(1)}, \beta^{(2)}, \ldots, \beta^{(p)}$ and standard deviation σ, simultaneously in the Phase II study. For the jth sample $(X, \boldsymbol{y}_j)$, they defined

$$\boldsymbol{Z}_j(\boldsymbol{\beta}) = (\hat{\boldsymbol{\beta}}_j - \boldsymbol{\beta})/\sigma \quad (5)$$

and