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A distance-based aggregation approach for group decision making with interval preference orderings ☆

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ABSTRACT

Xu (2013) proposed a nonlinear programming model to derive an exact formula to determine the experts' relative importance weights for the group decision making (GDM) with interval preference orderings. However, in this study, we show that the exact formula to determine the weight vector which always equals to $w = (1/m, 1/m, \dots, 1/m)^T$ (m is the number of experts). In this paper, we propose a distance-based aggregation approach to assess the relative importance weights for GDM with interval preference orderings. Relevant theorems are offered to support the proposed approach. After that, by using the weighted arithmetic averaging operator, we obtain the aggregated virtual interval preference orderings. We propose a possibility degree formula to compare two virtual interval preference orderings, then rank and select the alternatives. The proposed method is tested by two numerical examples. Comparative analysis are provided to show the advantages and effectiveness of the proposed method.

1. Introduction

Let a group decision making (GDM) problem defined by a finite set of n alternatives $X = \{x_1, x_2, \dots, x_n\}$, and a group of m experts $E = \{e_1, e_2, \dots, e_m\}$. In order to select the best alternative (s) or rank the alternatives, the decision maker (DM) needs to combine the individual experts' preferences into a group choice or consensus. There are many types of preference information which experts provide in GMD, such as ordinal ranking (Cook, 2006; Cook, Kress, & Seiford, 1996; Cook & Seiford, 1978, 1982), utility values (Chiclana, Herrera, & Herrera-Viedma, 1998; Herrera-Viedma, Herrera, & Chiclana, 2002; Wang, Yang, & Xu, 2005b), multiplicative preference relations (Saaty, 1980), fuzzy preference relations (Herrera-Viedma, Alonso, Chiclana, & Herrera, 2007; Kacprzyk, 1986; Tanino, 1984; Xu, 2004; Xu, Da, & Wang, 2010; Xu, Li, & Wang, 2014; Xu, Patnayakuni, & Wang, 2013a, 2013b; Xu & Wang, 2013), linguistic preference relations (Herrera & Herrera-Viedma, 2000; Herrera, Herrera-Viedma, & Verdegay, 1995; Xu, 2006; Xu & Wang, 2012b), intuitionistic fuzzy preference relations (Xu, 2007; Xu & Wang, 2012a), intuitionistic multiplication preference relations (Xia, Xu, & Liao, 2013), hesitant fuzzy preference relations (Liao & Xu, 2013; Xu & Liao, 2013), etc. Among the above preference information, ordinal data is an easy and effective way to express the decision makers' preferences. Many real world decision problems involve the use of ranking order or ordinal scale data. Ordinal data arise naturally in preferential election settings. Consider the situation in which each voter (expert) is requested to choose a subset of candidates from a ballot, and to rank order that subset from most to least preferred. Such a voting format is relatively common in municipal elections where a number of candidates are required to fill various positions (Cook, 2006). A complete ordinal ranking (no ties) of n alternatives must be an arrangement of the integers $\{1, \dots, n\}$. An important group of problems involving ordinal data and ranking concern the aggregation of preferences, provided by a set of experts, into a group preference function or a consensus. Many methods have been developed to aggregate ordinal preferences on a set of alternatives into a consensus, including Borda-Kendall method (Cook & Seiford, 1982), minimum variance method (Cook & Seiford, 1982), distance-based consensus models (Cook, 2006; Cook & Seiford, 1978; Cook et al., 1996), goal programming approach (González-Pachón & Romero, 1999), etc.

However, in sometime, the experts are indecisive, and thus give certain imprecision in their judgments. Consequently, the ranking

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given to the j th alternative by the experts is not a fixed number but a closed interval. Under this context, the experts may not give a complete ranking of alternatives but interval preference orderings (or called partial orders) (González-Pachón & Romero, 2001). For example, suppose that an investment company wants to invest a sum of money in the best option. There are four possible alternatives for the company to invest: a car company, a food company, a computer company and an arms company. The preferences of the company are as follows: The car company is ranked top 2, the food company is top 3, the computer company is second or third, the arms company is bottom 2. Such preferences can be represented easily and properly by interval preference orderings but cannot be depicted by any other structures (Fan & Liu, 2010). Generally, the decision may be made by a group of DMs. For the above example, the investment company has a group of departments, each department is directed by a DM, and each DM is an information source, and gives his/her interval ordinal preference orderings. Therefore, the GDM problem with interval ordinal preference orderings is an important research topic and received attention recently. González-Pachón and Romero (2001) proposed a method to aggregate the partial orders within a distance-based framework, and used the interval goal programming method to solve it. González-Pachón, Rodríguez-Galiano, and Romero (2003) used an interval goal programming method to deal with multi-criteria decision making problem with both interval preference orderings and pairwise comparison matrix where reciprocity and/or consistency are not verified. Fan and Liu (2010) proposed a possibility degree formula to compare two interval ordinal preference orderings, and then a collective expectation possibility degree matrix was built, and the optimization model was constructed to solve the GDM problem with interval preference orderings. All the above approaches to deal with GDM problems with exact ordinal data or interval preference orderings are assumed that all the experts have the same importance or the relative importance information of the experts is known. But this is sometimes unsuitable to depict the actual situations because the experts come from various research domains and do not have sufficient knowledge outside the scope of their domains, and hence they could not evaluate all aspects of the problem considered. Consequently, the individual experts necessitate different weights in the GDM process. In order to solve this problem, more recently, Xu (2013) established a nonlinear programming model by minimizing the divergences between the individual interval preference orderings and the group's options, from which he derived an exact formula to determine the experts' relative importance weights. However, as pointed out in Section 2 of this paper, the quadratic programming model has the drawback that the derived weights are always the same for each expert. This obeys his initial intention and makes the model redundant.

In this paper, we put forward a distance-based optimization model to derive the relative weights for experts in the GDM with interval preference orderings, then the linear weighted arithmetic averaging operator is used to obtain the collective virtual interval preference orderings. The proposed distance-based approach assesses the weights by minimizing the sum of squared distances between any two weighted interval preference orderings. The is the basic principle for generating an aggregated decision result.

The paper is organized as follows. Section 2 briefly reviews Xu (2013)'s method to derive the weights for group experts based on interval preference orderings, and some comments on drawbacks are pointed out. Section 3 develops a distance-based optimization model to determine the DMs' weights, then by using the weighted arithmetic averaging operator, we obtain the aggregated virtual interval preference orderings. We develop a possibility degree formula to compare two virtual interval preference orderings, then rank and select the alternative(s). In Section 4, two numerical examples are illustrated and comparative analysis are provided to show the advantages of the proposed method. This paper is concluded in Section 5.

2. A review of Xu's method for GDM model based on interval preference orderings

In this section, we introduce the notion of interval preference orderings which was provided by Fan and Liu (2010). And then we review Xu (2013)'s method for GDM model based on interval preference orderings.

Definition 1 Fan & Liu, 2010. Let Z^+ be the positive integer set. The interval preference ordering is expressed as $\tilde{r} = [r^-, r^- + 1, \ldots, r^+ - 1, r^+]$, where $r^-, r^+ \in Z^+, r^- \leqslant r^+$. r^- and r^+ are the lower and upper bounds of the ordinal interval preference ordering $\tilde{r}$. Particularly, if $r^- = r^+$, then the ordinal interval preference ordering $\tilde{r}$ is reduced to an exact preference ordering. For simplicity of representation, $\tilde{r} = [r^-, r^- + 1, \ldots, r^+ - 1, r^+]$ is denoted as $\tilde{r} = [r^-, r^+]$.

For a GDM problem, let $X = \{x_1, x_2, \ldots, x_n\}(n \geqslant 2)$ be a finite set of alternatives and $E = \{e_1, e_2, \ldots, e_m\}(m \geqslant 2)$ be a finite set of DMs, whose weight vector $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_m)^T$ is to be determined, where $\lambda_k \geqslant 0, k = 1, 2, \ldots, m, \sum_{k=1}^m \lambda_k = 1$. If all the experts are the same importance, then $\lambda_k = 1/m, k = 1, 2, \ldots, m$. However, in practical GDM problems, the experts may come from different research areas, and have distinct professional knowledge related to the problem domain, in these cases, the experts should be assigned different weights. The expert e_k provides his/her uncertain preferences on the set X, as a set of n interval preference orderings, $\widetilde{O}^{(k)} = \{\tilde{o}_1^{(k)}, \tilde{o}_2^{(k)}, \ldots, \tilde{o}_n^{(k)}\}$, where $\tilde{o}_i^{(k)} = \left[o_i^{-(k)}, o_i^{+(k)}\right]$ represents an interval-valued preference ordering given by the expert e_k to the alternative x_i, each $\tilde{o}_i^{(k)}$ consists of a collection of positive integers ranked in increasing order. For example, $\tilde{o}_i^{(k)} = [1, 3]$ represents the possible ranking ordinals of an alternative of x_i given by the expert e_k may be first, second and third. It is naturally assumed that the smaller $\tilde{o}_i^{(k)}$, the better the alternative x_i.

To obtain the collective option for the group, Xu (2013) employed the Weighted Arithmetic Averaging (WAA) operator:

$$\tilde{o}_i = \left[o_i^-, o_i^+\right] = \sum_{k=1}^m \lambda_k \tilde{o}_i^{(k)} = \left[\sum_{k=1}^m \lambda_k o_i^{-(k)}, \sum_{k=1}^m \lambda_k o_i^{+(k)}\right], \quad \text{for all } i = 1, 2, \ldots, n \tag{1}$$

to aggregate individual interval preference orderings $\widetilde{O}^{(k)} = \left\{\tilde{o}_1^{(k)}, \tilde{o}_2^{(k)}, \ldots, \tilde{o}_n^{(k)}\right\}(k = 1, 2, \ldots, m)$ into a set of collective interval preference orderings $\widetilde{O} = \{\tilde{o}_1, \tilde{o}_2, \ldots, \tilde{o}_n\}$, i.e., $o_i^- = \sum_{k=1}^m \lambda_k o_i^{-(k)}$ and $o_i^+ = \sum_{k=1}^m \lambda_k o_i^{+(k)}, i = 1, 2, \ldots, n$.

Clearly, a key issue in applying the WAA operator is to determine the weight vector λ. If each individual's interval preference orderings are consistent with the collective interval preference orderings, then $\widetilde{O}^{(k)} = \widetilde{O}$, i.e.,

$$o_i^{-(k)} = \sum_{l=1}^m \lambda_l o_i^{-(l)}, o_i^{+(k)} = \sum_{l=1}^m \lambda_l o_i^{+(l)}, \quad \text{for all } i = 1, 2, \ldots, n, \quad k = 1, 2, \ldots, m \tag{2}$$

However, Eq. (2) generally does not hold. Thus, Xu (2013) introduced a deviation variable $\tilde{d}_i^{(k)}$ as:

$$\tilde{d}_i^{(k)} = \left(o_i^{-(k)} - \sum_{l=1}^m \lambda_l o_i^{-(l)}\right)^2 + \left(o_i^{+(k)} - \sum_{l=1}^m \lambda_l o_i^{+(l)}\right)^2, \quad \text{for all } k = 1, 2, \ldots, m, \ i = 1, 2, \ldots, n \tag{3}$$

and constructed a quadratic deviation model: