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Prioritized multi-criteria decision making based on preference relations ☆

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ABSTRACT

There may exist priority relationships among criteria in multi-criteria decision making (MCDM) problems. This kind of problems, which we focus on in this paper, are called prioritized MCDM ones. In order to aggregate the evaluation values of criteria for an alternative, we first develop some weighted prioritized aggregation operators based on triangular norms (*t*-norms) together with the weights of criteria by extending the prioritized aggregation operators proposed by Yager (Yager, R. R. (2004). Modeling prioritized multi-criteria decision making. *IEEE Transactions on Systems, Man, and Cybernetics, 34*, 2396–2404). After discussing the influence of the concentration degrees of the evaluation values with respect to each criterion to the priority relationships, we further develop a method for handling the prioritized MCDM problems. Through a simple example, we validate that this method can be used in more wide situations than the existing prioritized MCDM methods. At length, the relationships between the weights associated with criteria and the preference relations among alternatives are explored, and then two quadratic programming models for determining weights based on multiplicative and fuzzy preference relations are developed.

1. Introduction

Since it was introduced in mid-1960s, multi-criteria decision making (MCDM) has become an important part of decision sciences (Koele, 1995; Yager & Rybalov, 1998; Zhang, Liu, & Zhai, 2011). It is mainly used to prescribe ways of evaluating, ranking and selecting the most favorable alternative(s) from a set of available ones which are characterized by multiple and usually conflicting criteria. However, the criteria are not always independent in some actual MCDM problems, and a possible relationship between a pair of criteria is the prioritization (Chen & Wang, 2009; Wang & Chen, 2007; Yan, Huynh, Nakamori, & Murai, 2011; Yager, 2004, 2008, 2009; Yager, Walker, & Walker, 2011). A typical example concerns the criteria of *safety* and *cost* in the cases of buying a car (Yan et al., 2011), selecting a bicycle for child (Yager, 2008) or air travel (Yager, 2004), etc. In the cases, we usually do not allow a loss in *safety* to be compensated by a benefit in *cost*, i.e., tradeoffs between *safety* and *cost* are unacceptable. Simply speaking, the criterion *safety* has a higher priority than *cost*. Moreover, according to Yager's (2008) results, there may exist priority relationships among criteria in the problems of information retrieval (Herrera-Viedma, Pasi, López-Herrera, & Porcel, 2006). For example, a user intends to look for literature about *decision making* and prefers if they were written *after 2003*. In this case, the condition about *decision making* has a priority, because the user will not be interested if a paper (or book) is not about the topic. It can be expressed by a linguistic formulation, "the user wants the criterion *decision making*, and if possible he/she also wants another criterion *after 2003*". This kind of MCDM, in which there are priority relationships among criteria, is called prioritized MCDM.

Recently, the research about the prioritized MCDM problems has focused on generating or devising weights associated with criteria for common aggregation operators according to the priority relationships on the basis of consensus, in which the importance weights associated with the lower priority criteria are related to the satisfactions of the higher priority criteria. For example, Yager (2004) introduced an ordered weighted averaging (OWA) prioritized criteria aggregation, in which the weight of a criterion is determined by the original OWA weighting vector together with the satisfactions of the criteria with higher priority. According to Wang & Chen (2007) and Chen & Wang (2009), the requirements of criteria are introduced to describe the influence of the priority relationships, and the weights of the lower priority criteria of each alternative depend on whether each alternative satisfies the

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requirements of all the higher priority criteria or not. In the process of handling the prioritized multi-criteria aggregation problems with weakly ordered prioritizations, Yager (2008) designed several approaches to derive the overall satisfactions associated with the higher prioritized hierarchies (by means of the min operator and the OWA operator, etc.), based on which the weight of each hierarchy can be calculated, and then a prioritized "anding" operator and a prioritized "oring" operator were introduced based on triangular norms and conorms respectively. The prioritized multi-criteria aggregation problems were solved with strictly ordered prioritizations on the basis of the OWA operator by Yager (2009). A monotonic set measure was used to describe the priority relationships by Yager et al. (2011), and then an integral type aggregation (Choquet integral) was used to aggregate the evaluation values of criteria. Finally, Yan et al. (2011) proposed a prioritized weighted aggregation operator based on the OWA operator along with triangular norms, and furthermore, considering the decision maker's requirements toward the higher priority hierarchy, a benchmark based approach was designed to induce the priority weight for each priority hierarchy. Some existing methods have been validated to be imperfect. For example, according to Wang & Chen (2007) and Chen & Wang (2009), there have been some drawbacks in Yager's (2004) method, including the inappropriate use of quantifier-guided aggregation and the incorrect results in some situations, with the help of some numerical examples, and then proposed their prioritized aggregation (PA) operator. Besides, Yan et al. (2011) pointed out some limitations and drawbacks in Yager (2004, 2008), Wang & Chen (2007) and Chen & Wang (2009): (1) tradeoffs of criteria in the same prioritized hierarchy is identical if the PA operators in Yager (2004, 2008) are used to handle the prioritized MCDM and (2) the satisfaction function (see (11)) proposed by Wang & Chen (2007) and Chen & Wang (2009) is too strict for decision making under prioritized environment. Therefore, it is still meaningful to improve and develop the methods of prioritized MCDM up to now. After investigating previous works in detail, we discover two imperfectnesses remained in them:

(1) The previous methods are inapplicable in a special situation that the evaluation values of all alternatives with respect to the criterion with the highest priority are close and do not satisfy the requirement of the decision maker. For example, suppose the *safety* indices of alternative cars are the same and do not satisfy the requirement of the consumer, then any car, but not the cheapest one, can be selected if any one of previous methods are used to solve this problem.
(2) Weights of criteria are associated with the priority relationships closely as stated by previous works including Yager (2004), Wang & Chen (2007), and Chen & Wang (2009), etc. As is well-known, lots of methods, just like preference relations, contribute themselves to obtain the weights by well using some given information. The relationships between such kinds of information and the given priority relationships among criteria are not clarified in any previous work. More importantly, because the priority relationships and some other given information impact the weights more or less, it is usually unwise of us to tackle different kinds of information asynchronously in some practical prioritized MCDM problems.

Motivated by the above analysis, in this paper, we consider the situations where the weights associated to criteria are given during the prioritized MCDM, and show how to revise the weights associated to the lower priority criteria concerning the satisfaction degrees of higher priority criteria, based on which we develop some weighted prioritized aggregation operators by extending the prioritized aggregation operators proposed by Yager (2004). Sequentially, we notice a kind of situations that the influences from the higher priority criteria to the lower priority ones are related to the concentration degrees of the evaluating values with respect to the higher priority criteria. A special example is that the *safety* indices of alternative cars are the same. In this case, no matter whether the *safety* indices satisfy the consumer or not, he/she will select the cheapest car. In another word, there is no influence from *safety* with higher priority to *price* with the lower priority in the example, because of the extremely concentrative evaluating values of *safety* for all alternative cars. Therefore, when handling a prioritized MCDM problem, we must realize the more concentrative the evaluation values of a criterion, the less influence to its lower priority criteria, based on which we design a more proper model of the prioritized MCDM. Compared to the existing prioritized MCDM method based on the prioritized aggregation operators (Chen & Wang, 2009; Wang & Chen, 2007; Yager, 2004; Yan et al., 2011), our method is validated to be more useful and feasible in the above special situation. Furthermore, how to determine the weights according to other given information is concerned in the prioritized MCDM problems in this paper. Some practical models for weights determination based on two kinds of preference relations are developed. Then the weights can be easily determined according to the preference relations of pairs of alternatives by solving the corresponding quadratic programming problems.

The rest of the paper is organized as follows: We first introduce some basic concepts and terminologies in Section 2. Then on the basis of the prioritized aggregation operators, Section 3 proposes two kinds of the weighted prioritized aggregation operators with respect to strictly and weakly ordered prioritizations among criteria. In Section 4, a prioritized MCDM model is developed based on the weighted prioritized aggregation operators, a car selecting example is taken to illustrate our prioritized MCDM method, and some advantages and limitations of our method are discussed. Furthermore, in a prioritized MCDM problem, how to determine weights in accordance with multiplicative and fuzzy preference relations respectively is discussed in Section 5, and two corresponding quadratic programming models are proposed. At length, the conclusions are given in Section 6.

2. Preliminaries

In this section, we will introduce some basic concepts and terminologies, such as multi-criteria decision making (MCDM), aggregation functions, priority relationships, and preference relations and so on, for the purpose of their use in the following sections.

2.1. Multi-criteria decision making (MCDM) and aggregation functions

The fundamental components of a MCDM problem are a set of criteria, $C = \{c_1, c_2, \ldots, c_n\}$, of interest to the decision maker and a set of possible alternatives, $X = \{x_1, x_2, \ldots, x_m\}$, so as to evaluate each alternative and select the best one(s). In their pioneering work on MCDM, Bellman & Zadeh (1970) suggested that each criterion can be represented as a fuzzy subset over the alternatives. In particular, if c_j ($j = 1, 2, \ldots, n$) is a criterion, then we can represent it as a fuzzy subset c_j over X such that $c_j(x_i)$ is the degree to which this criterion is satisfied by the alternative x_i, i.e., $c_j(x_i)$ is the evaluation value of the criterion c_j over x_i. Here, we shall assume $c_j(x_i) \in [0, 1]$ ($i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$). In this case, for each alternative x_i, we can always utilize an aggregation function AF to aggregate all relevant evaluation values $c_j(x_i)$ ($j = 1, 2, \ldots, n$) into an overall evaluation value $c(x_i)$, i.e.,

$$c(x_i) = AF(c_1(x_i), c_2(x_i), \ldots, c_j(x_i)) \tag{1}$$