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A variable-reduction technique for the fixed-route vehicle-refueling problem ☆

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ABSTRACT

The *fixed-route vehicle-refueling problem* (FRVRP) is a difficult combinatorial problem that is used extensively in the US truckload industry to manage fuel costs. This paper proposes a preprocessing technique for the FRVRP that cuts the problem size noticeably without eliminating the optimal solution(s), which allows users to enlarge the size of solvable instances or save the CPU time of solving the problem dramatically. Empirical testing with real-world instances shows that our method: (*i*) reduces the problem size by 54.8% and (*ii*) solves the FRVRPs to optimality in roughly 1/4 of the time it is currently taking.

1. Introduction

Given the recent trend of raising fuel cost, the efficient management of fuel cost has become an important issue for all the modes of transportation (see, e.g., Kuo, 2010). In this paper we consider a fuel-management decision problem widely used in the truckload industry, called the *fixed-route vehicle-refueling problem* (FRVRP). The FRVRP is a combinatorial problem which seeks the best *refueling policy* (sequence of fuel stations to use, along with the best refueling quantity at each station) for a given origin–destination route that minimizes a vehicle's refueling cost. Although the FRVRP is of interest in its own right, it is known to have a strong relationship with the *capacitated lot-sizing problem* (CLSP) (e.g., Atamturk & Kucukyavuz, 2005; Love, 1973), and has also received attention from the researchers working in this area. The FRVRP is studied extensively by practitioners too. Consulting companies have developed a class of software products called *fuel optimizers* for truckload carriers that solve the FRVRP to near optimality. These products, which typically work with truck-routing software, first compute the shortest route for a given origin–destination, and then compute the refueling policy for this route by using the latest (updated daily) fuel price data obtained from the *OPIS* (Oil Price Information Service) database.

There are several variants, or extensions, of the FRVRP, most of which address the vehicle-routing and the vehicle-refueling problems jointly. These variants, which are often called the *variable-route vehicle-refueling problem* (VRVRP), include the problems proposed by (*i*) Suzuki and Dai (2012), which jointly considers the shortest-route problem (SRP) and the FRVRP, (*ii*) Suzuki (2012), which jointly addresses the traveling salesman problem with time windows (TWPTW) and the FRVRP, (*iii*) Bousonville, Hartmann, Melo, and Kopfer (2011), which jointly tackles the vehicle-routing problem with time windows (VRPTW) and the FRVRP, and (*iv*) Sweda and Klabjan (2012), which jointly considers the SRP and the FRVRP for electric vehicles. From the perspective of researchers studying these variants, the FRVRP is a sub-problem of their focal problems (e.g., as part of "route first, refueling-policy second" approaches to the VRVRP), which suggests that the FRVRP solution methods can be of great utility to these researchers too.

1.1. Types of FRVRP

To date, two types of FRVRP have been considered in the literature. The first, and the simpler, version makes the following assumptions: (*i*) every fuel station (truck stop) is located "on the route" so that the travel distance of a vehicle from origin to destination is unaffected by the choice of truck stops and (*ii*) refueling quantity at each truck stop is not subject to a specific lower bound (but must be non-negative). We denote this type of FRVRP as *basic FRVRP*. The basic FRVRP can be viewed as a special case of CLSP where we assume a fixed inventory capacity, zero setup cost, zero inventory cost, and linear production cost function (Lin, Gertsch, & Russell, 2007). The basic FRVRP can be formulated as a pure network optimization problem, which can be easily solved to

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optimality by using the standard linear-programming techniques. Efficient exact methods that are designed specifically to solve the basic FRVRP are also available. Lin et al. (2007) proposed an $O(n)$ greedy algorithm, where n is the number of truck stops available between origin and destination, and Khuller, Malekian, and Mestre (2008) proposed an $O(n \log n)$ algorithm.

The second version relaxes the two assumptions used in the basic FRVRP; i.e., it considers both the *minimum refueling quantity* ($\rho \geqslant 0$), and the *out-of-route miles* for each truck stop ($e_i \geqslant 0$, $i = 1, 2, \ldots, n$; which reflects the distance a truck must divert from the primary route to reach station i). We denote this type of FRVRP as *complex FRVRP*. The complex FRVRP is a generic form of the basic FRVRP, as the latter can be viewed as a special case of the former where $\rho = 0$ (or ρ = theoretical lower bound of the refueling quantity per stop implied by the parameters) and $e_i = 0 \; \forall \; i = 1, 2, \ldots, n$. The significance of considering ρ and e_i are as follows. First, ρ allows users to not only control the frequency of fuel stops, but also give truck drivers some benefits at truck stops (e.g., many truck stops give free showers, free overnight parking, or discount meal coupons to drivers who buy 50 gallons or more). Second, e_i affects the "attractiveness" of truck stops such that the larger the e_i, the less attractive the truck stop i (e.g., if $e_i > e_j$, i may be less attractive than j even if i has cheaper price than j, because a vehicle must burn more fuel to reach i than to reach j). This second condition implies that, if we ignore e_i in an FRVRP, we may find *sub-optimal* solutions (it is worth noting that $e_i > 0$ for most fuel stations found on major US highways).

Since ρ and e_i are important parameters that control both the quality of solutions and the "driver satisfaction" with the policies, all commercial fuel optimizers use the complex FRVRP. Our interviews with four US truckload carriers (all of which are fuel-optimizer users) also indicate that they strongly prefer the complex FRVRP to the basic FRVRP for the following reasons. First, they prefer to use ρ of at least 50 gallons to make their drivers happy (give benefits at truck stops), so that they can attain high rate of driver compliance to refueling instructions (see Suzuki (2009), for similar claims). Second, they believe that, typically, cheap truck stops are located "off" the major highways ($e_i > 0$), so that if we ignore e_i the resulting solution may tend to choose the stations with large e_i values, which is not only sub-optimal but also inconvenient for truck drivers. Since all of the above conditions suggest that the complex FRVRP is preferred to the basic FRVRP, we focus on discussing the complex FRVRP in the remainder of this paper.

1.2. Current approaches to complex FRVRP

The complex FRVRP has been explored by a limited number of studies to date. The complex FRVRP can be viewed as a single-item CLSP with a fixed production capacity, fixed minimum production lot-size, fixed inventory capacity, non-negative and non-stationary setup cost, zero inventory cost, and linear (but time-varying) production cost function. The single-item CLSP with set up costs itself is already NP-hard (Karimi, Fatemi Ghomi, & Wilson, 2003; Okhrin & Richter, 2011), and both the inventory capacity and the lower bound in lot-sizing add extra complexities to the problem (the added complexities of including inventory bounds in CLSP are well documented in Atamturk and Kucukyavuz (2005), and those of including the minimum lot size in CLSP are discussed in Constantino (1998) and Okhrin and Richter (2011)). These conditions suggest that it may be difficult to develop efficient optimal algorithms for the complex FRVRP. Although some efficient algorithms exist that can solve the CLSP with inventory bounds (e.g., Gutierrez, Sedeno-Noda, Colebrook, & Sicilia, 2002) or the CLSP with minimum lot sizes (e.g., Okhrin & Richter, 2011), we are not aware of any efficient algorithm that can solve the CLSP with both the inventory bound and the minimum lot size, along with the setup cost, production capacity, and time-varying production costs.

Currently, two approaches are available for solving the complex FRVRP. The first is to use heuristic methods (all fuel optimizers employ this approach).[1] The major limitation of this approach is that it may not produce quality solutions. This approach also seems to suffer from the limited size of solvable instances, as most fuel optimizers cannot handle large problems (possibly because of memory limitations or restrictions on running time). *ProMiles*, for example, one of the most-widely used fuel optimizers in the field, cannot solve instances for which $n > 250$. The second approach is to use exact methods. To the best of our knowledge, the only study that proposed an exact method for the complex FRVRP is Suzuki (2008), which used a mixed-integer linear programming approach. This method, however, relies on a branch-and-bound technique, so that its solution time grows exponentially with the problem size (n). This means that, perhaps, the method cannot solve large FRVRP instances efficiently.

The above paragraph indicates that the existing techniques for the complex FRVRP suffer from either: (*i*) solution quality and the limited size of solvable instances (heuristics), or (*ii*) relatively long CPU time when n is large (exact method). This is the major point of concern for many truckload carriers, as they are frequently required to solve large complex FRVRPs with more than 400 truck stops within very narrow time windows. This issue becomes even more important in the future, because the practical size of the FRVRP is growing rapidly. As a result of the advancement in information technology, carries now have more accurate data on future load tenders, which allows them to incorporate two or more loads into one refueling problem to generate better solutions (e.g., if we know that a truck going to point A will visit point B subsequently, we can derive a better FRVRP solution by solving the combined problem that considers both legs; see. e.g., Suzuki, 2009). This suggests that, currently, the existing complex FRVRP methods, none of which can handle large instances efficiently, may be providing limited practical values.

1.3. Proposed approach and study goal

One way to efficiently solve large complex FRVRPs is to develop a preprocessing technique that allows us to reduce the problem size (n), prior to solving the problem, by removing those truck stops that are guaranteed not to be chosen as the refueling points by the optimal policy. This approach has several interesting features from the practical standpoint. First, since this technique merely reduces the problem size, it can be combined with *any* existing method, heuristic or exact, to cut the computational effort of solving the complex FRVRP (it can also be combined with any basic FRVRP technique, since the complex FRVRP is a generic form of the basic FRVRP). Second, this technique can be used in conjunction with commercial fuel optimizers to help these products solve large problems that are currently beyond their capabilities. Third, if combined with fuel optimizers, this technique may possibly improve (in some instances) the solution quality of these products because, given that it removes only the "unpromising" part of the feasible region, the use of this technique should not worsen the solution quality.

Given these features this paper develops a preprocessing technique that allows us to reduce truck stops in an FRVRP substantially without eliminating the optimal solution(s). Development of such a technique has significant practical values. First, it allows

[1] It is not clear how the FRVRP is solved by fuel optimizers, as fuel-optimizer vendors are reluctant to provide the details of their solution methods (we contacted multiple software vendors, but they all refused to provide the details). We suspect that they may be using a simple method in which they first identify the set of cheapest truck stops along the shortest route, and then choose a few truck stops from this set by using a construction-type heuristic.