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Computational performances of a simple interchange heuristic for a scheduling problem with an availability constraint ☆

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ABSTRACT

This paper deals with a scheduling problem on a single machine with an availability constraint. The problem is known to be NP-complete and admits several approximation algorithms. In this paper we study the approximation scheme described in He et al. [Y. He, W. Zhong, H. Gu, *Improved algorithms for two single machine scheduling problems*, Theoretical Computer Science 363 (2006) 257–265]. We provide the computation of an improved relative error of this heuristic, as well as a proof that this new bound is tight. We also present some computational experiments to test this heuristic on random instances. These experiments include an implementation of the fully-polynomial time approximation scheme given in Kacem and Ridha Mahjoub [I. Kacem, A. Ridha Mahjoub, *Fully polynomial time approximation scheme for the weighted flow-time minimization on a single machine with a fixed non-availability interval*, Computers and Industrial Engineering 56 (2009) 1708–1712].

1. A single machine total completion time scheduling problem with an availability constraint

Scheduling jobs under maintenance constraints is an important issue in many real-life situations. For instance, Seristö (1995) claims that maintenance represents between 10% and 15% of the total operating expenses of airlines. Recall that, for an airline like Lufthansa Group or Air France–KLM, the yearly total operating expenses lied around 30 billions US dollars in 2009 (World Airline Report, 2010). More generally, efficiently scheduling jobs under availability constraints (due to, e.g., maintenance) is a challenge which is often motivated by consequent financial stakes. Besides, scheduling problems with availability constraints are widely studied in the literature (see for instance (Lee, 2004; Sanlaville & Schmidt, 1998; Schmidt, 2000) for surveys), and is an active area of research.

In this paper, we consider the problem of scheduling jobs on a single machine having one period of maintenance. This period of maintenance is known in advance, and is such that no job can be done during it. In other words, preemption is not allowed, and the machine is not available for processing jobs during the maintenance. We wish to minimize the total completion time of the jobs. Since the period of maintenance is known in advance, this problems models also other situations where the machine is unavailable besides maintenance.

This particular problem is usually denoted $1, h_1/\sum C_i$. Adiri, Bruno, Frostig, and Rinnooy Kan (1989) and Lee and Liman (1992) showed that this problem is NP-hard. Lee and Liman also showed that the SPT heuristic, which consists in sorting the jobs in non-decreasing order of their processing times, leads to a heuristic of relative error $\frac{2}{7}$.

Sadfi, Penz, Rapine, Błażewicz, and Formanowicz (2005) proposed an improved heuristic for this problem, having a relative error of $\frac{3}{17}$. Their heuristic is a post-optimization of SPT using a 2-OPT procedure. More precisely, let us denote A and B the sets of jobs scheduled respectively after and before the maintenance by the SPT algorithm. The heuristic consists in exchanging one job of A with one job of B in order to improve the total completion time. They call their procedure MSPT, for Modified SPT.

In He, Zhong, and Gu (2006) the authors study a generalization of MSPT, that we call here MSPT-k. This heuristic consists in exchanging at most k jobs of A with at most k jobs of B, with k a fixed positive constant. In He et al. (2006) they prove that for all $k \geqslant 2$, MSPT-k has a relative error bounded by

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$$a_k = \frac{2}{5 + 2\sqrt{2k+8}}.$$

Breit (2007) gives a $O(n \log n)$ algorithm having an error bound of 1.074. In Kacem and Ridha Mahjoub (2009) provide an FPTS (fully polynomial time approximation scheme) for the weighted version of the problem, namely $1, h_1//\sum w_i C_i$. This FPTAS uses as a subroutine a 2-approximation algorithm of Kacem (2008). Another FPTAS is given in Kellerer and Strusevich (2010), whose running time is dominated by the one of Kacem and Ridha Mahjoub.

There exist several other papers dealing with the problem in the literature. In particular, the preemptive case has been recently studied (Kacem & Chu, 2008c; Kellerer & Strusevich, 2010; Wang, sun, & Chu, 2005), as well as other variants (Kacem & Kellerer, 2011; Mellouli, Sadfi, Chu, & Kacem, 2009; Tan, Chen, & Zhang, 2011) including the weighted version of the problem (Kacem & Chu, 2008a, 2008b; Kacem, Chu, & Souissi, 2008; Kellerer, Kubzin, & Strusevich, 2009).

In this paper, we prove in Section 3 that for all $k \geqslant 2$, MSPT-k has a relative error bounded by

$$\alpha_k = \frac{k+2}{2k^2 + 8k + 7}.$$

Since $\alpha_k < a_k$, this improves the result of He et al. (2006). In addition, we prove in Section 4 that this relative error is tight, in the sense that we give a construction of a family of instances such that, asymptotically, the relative error of MSPT-k applied to these instances is α_k. Our approach thus generalizes and unifies the results of He et al. (2006), Lee and Liman (1992), and Sadfi et al. (2005).

We then present the results of the computational experiments we carried out to test MSPT-k on some randomly generated instances. These computational experiments include an implementation of the FPTAS of Kacem and Ridha Mahjoub (2009).

2. Notations

Let $J = \{J_i | i = 1, \ldots, n\}$ be the set of jobs. We use the following notations, which, for convenience, are the same as the ones used in Sadfi et al. (2005):

$J_{[i]}$	Job scheduled at position i
p_i	Processing time of job J_i
$p_{[i]}$	Processing time of job scheduled at position i
C_i	Completion time of job J_i
$C_{[i]}$	Completion time of job scheduled at position i
R	Starting time of maintenance
L	Duration of maintenance
D	Ending time of maintenance (hence $D = R + L$)
δ	Idle time of the machine before the maintenance

The MSPT-k heuristic is the following:
MSPT-k heuristic

(1) Schedule the jobs according to the SPT rule.
(2) Denote A the set of jobs scheduled after the maintenance, and B the set of jobs scheduled before.
(3) Try all possible exchanges of at most k jobs of A with at most k jobs of B (the jobs before and after the maintenance being scheduled in non-decreasing order of their processing times).
(4) Output the best exchange found in step 3.

Note that MSPT-0 is just the SPT algorithm, and MSPT-1 is the MSPT heuristic of Sadfi et al. (2005). In He et al. (2006) they use the notation SPTE to denote MSPT-k, but here we prefer the use of the notation MSPT-k to make the dependence in k explicit.

3. An improved relative error bound for MSPT-*k*

In this section we derive some properties of the solutions obtained using SPT and MSPT-k, for a fixed $k \geqslant 2$. The following lemmata enable us to analyze the MSPT-k heuristic and to compute an improved error bound.

We will use the following notations, which are the same as the ones used in Sadfi et al. (2005): The schedule generated by the SPT algorithm will be denoted S, the optimal schedule will be denoted S^*, and the schedule generated by MSPT-k will be denoted S'. Clearly, any schedule can be seen as a partition of the jobs into two sets: Those which are scheduled before the maintenance, and those which are scheduled after the maintenance. Indeed, once the partition of jobs is fixed, it is dominant to schedule them in non increasing order of their processing time.

Let A be the set of jobs scheduled after the maintenance in S, and let B be the set of jobs scheduled before the maintenance in S. With straightforward notations, A' and B' represents the job partition in S'. Now, let X be the set of the $|B|$ first jobs scheduled in S^*, and let Y be the set of remaining jobs (note that $|Y| = |A|$).

Note that MSPT-k exchanges t jobs of A with t' jobs of B, with $t \leqslant k$, $t' \leqslant k$, and $t' \geqslant t$. Hence we have $|A'| \geqslant |A| = |Y|$ and $|B'| \leqslant |B| = |X|$.

Finally, let us denote C_i, C'_i, and C^*_i the completion times of job J_i in schedules S, S', and S^* respectively. Since MSPT-k clearly improves SPT, then we have $\sum_i C'_i \leqslant \sum_i C_i$. Let us also denote $J'_{[i]}$ and $J^*_{[i]}$ the jobs scheduled at position i in S' and S^*, respectively. The straightforward notations $p'_{[i]}$, $p^*_{[i]}$, $C'_{[i]}$ and $C^*_{[i]}$ will also be used in the sequel. Throughout this section we consider the case where $|A| \geqslant k+1$ and $|B| \geqslant k+1$, because if not, then MSPT-k is clearly optimal.

3.1. Preliminary lemmata

This lemma generalizes Lemma 1 in Sadfi et al. (2005), that stated the result only for the case $S = S'$ and $S = S^*$. Here we restate the lemma for any schedule S which is better than S. The proof being exactly the one of Sadfi et al. (2005), it is omitted here.

Lemma 1 (*Sadfi et al.*). *Let S be a schedule better than S, that is to say its total completion time is better that the one of S. Let us denote δ and δ the idle time of the machine before the maintenance in S and S, respectively. Then we have $\delta \leqslant \delta$.*

The following lemma is Lemma 2 of Sadfi et al. (2005), that we recall here without any proof.

Lemma 2 (*Sadfi et al.*). *Let $C_{[i]}$ and $C^*_{[i]}$ be the completion times of the job scheduled at position i in the SPT and in the optimal solution, respectively. Then we have:*

$$\sum_{J_{[i]} \in A} C_{[i]} \leqslant \sum_{J_{[i]} \in Y} C^*_{[i]} + |Y|(\delta - \delta^*).$$

The following lemma generalizes Lemma 3 of Sadfi et al. (2005) and Lemma 11 of He et al. (2006).

Lemma 3. *Let $t \geqslant 1$ be an integer. If (at least) t jobs of X are scheduled after the period of maintenance in the optimal solution, then we have:*

$$\sum_{i=1}^{n} C'_i \leqslant \sum_{i=1}^{n} C^*_i + (|Y| - (t+1))(\delta - \delta^*).$$

Proof. Since at least t jobs of X are scheduled after the period of maintenance in the optimal solution, then we have:

$C^*_{[i]} \geqslant C_{[i]}$ for all $i = 1, \ldots, |B| - t$,