Computers & Industrial Engineering 63 (2012) 763–778

Contents lists available at SciVerse ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Nonlinear fixed charge transportation problem by minimum cost flow-based genetic algorithm

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ARTICLE INFO

Article history:
Received 23 September 2011
Received in revised form 5 January 2012
Accepted 29 April 2012
Available online 7 May 2012

Keywords:
Genetic algorithms
Nonlinear fixed charge problem
Transportation problem
Minimum cost flow
Network with lower & upper arc capacities

ABSTRACT

Nonlinear Fixed Charge Transportation Problem (NFCTP) is a variant of fixed charge transportation problem, which is to ship available amounts of goods to satisfy the demands at minimal total cost, on condition that any route has a fixed cost irrelative to its shipping amount if it is used, and a variable cost directly proportional to the quadratic of its shipping amount as a nonlinear term. This paper aims at developing an efficient method to solve NFCTP. In this paper, NFCTP is formulated using a mixed integer programming model. Based on steady-state genetic algorithm as framework, and minimum cost flow algorithm as decoder, a hybrid genetic algorithm named *NFCTP-HGA* is proposed as a solution method of the model. Taking advantage of nonlinear structure and special network structure of NFCTP, the *NFCTP-HGA* algorithm has good performance in the sense of being implemented on computer, computational time, required memory for computation, and ability to search global optimum. The application of the *NFCTP-HGA* algorithm is illustrated with examples. Numerical experiments demonstrate that the *NFCTP-HGA* algorithm is an efficient and robust method to solve NFCTP, especially applicable to large scale problems.

1. Introduction

Networks provide a useful way for modeling real world problems, and are extensively used in logistics, communications, as well as hydraulic, mechanical, and electronic systems. As is well known, as an important network problem, *Transportation Problem* (TP) can be formulated and solved as a linear programming. Since Hitchcock (1941) first developed the transportation model, TP has been studied from different standpoints by a number of scholars, such as Arsham (1992), Arsham and Khan (1989), Adlakha and Kowalski (1998, 2000, 2001), Brenner (2008), Charnes and Copper (1954), Dantzig (1963), and Liu (2003).

As an extension of TP, Fixed Charge Transportation Problem (FCTP) is to ship available amounts of goods to satisfy the demands at minimal total cost, on condition that any route has a fixed cost irrelative to its shipping amount if it is used, and a variable cost proportional to its shipping amount. FCTP is a NP-hard problem (Hirsch and Dantzig, 1968; Klose, 2008), and it can be formulated and solved as a mixed integer programming (Balinski, 1961). Not having taken advantage of special network structure of FCTP, general mixed integer programming solution methods such as branch-and-bound method, cutting plane method, are generally inefficient and computationally expensive (Steinberg, 1970). Therefore, a number of scholars, such as Adlakha and Kowalski (1999, 2003), Adlakha et al. (2007), Sun et al. (1998), and Xie and Jia (2009a) (for details, see the literature review in this paper), have given solution procedure for FCTP (Kowalski and Lev, 2008). It is especially worthy to be mentioned that Gen et al. (2000) proposed spanning tree-based genetic algorithm for FCTP using Prüfer number encoding.

As a variant of FCTP, Nonlinear Fixed Charge Transportation Problem (NFCTP) is to ship available amounts of goods to satisfy the demands at minimal total cost, on condition that any route has a fixed cost irrelative to its shipping amount if it is used, and a variable cost directly proportional to the quadratic of its shipping amount as a nonlinear term. NFCTP is also a NP-hard problem, and it is more difficult to resolve than FCTP due to the existence of nonlinear terms. Based on the work of Gen et al. (2000) and Jo et al. (2007) developed a spanning tree-based genetic algorithm to tackle NFCTP using Prüfer number representation; and they mentioned that the spanning tree-based genetic algorithm should be further improved while used to solve NFCTP due to not having absorbed the characteristic of nonlinear structure.

Having important application value in real world systems, NFCTP has attracted the attention of a number of scholars since it was considered by Jo et al. (2007). On one hand, as further development of the work of Jo et al. (2007) and Hajiaghaei-Keshteli

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http://dx.doi.org/10.1016/j.cie.2012.04.016

et al. (2010) improved the spanning tree-based genetic algorithm by presenting a pioneer method to design a chromosome that does not need a repairing procedure for feasibility. On the other hand, Kannan et al. (2008) made comments on calculation of total cost as well as indication of problem size for NFCTP, and declared that they found an error in calculating the total transportation costs of the two previously addressed problems by Jo et al. (2007). As a response to the comments of Kannan et al. (2008), Xie and Jia (2010) illustrated the formula for calculating the total cost of NFCTP with the two previously addressed problems by Jo et al. (2007), and found better near-optimal solutions for the two problems.

The improved spanning tree-based genetic algorithm by Hajiaghaei-Keshteli et al. (2010) has taken advantage of special network structure, but it has not yet absorbed the characteristic of nonlinear structure of NFCTP (Gen et al., 2000; Jo et al., 2007). Therefore, while such spanning tree-based genetic algorithm is used to solve NFCTP, it is still possible that the quality of the found solution is not too ideal. Our goal is to develop an efficient method to solve NFCTP by means of taking advantage of nonlinear structure and special network structure of NFCTP.

In order to attain our goal, we state NFCTP as follows (Dahiya and Verma, 2007).

There are õ suppliers (origins) such as $O_i (i \in I = \{1, 2, \ldots, \tilde{o}\})$, which can supply a kind of goods for ñ customers (destinations) such as $D_j (j \in J = \{1, 2, \ldots, \tilde{n}\})$. The following parameters are known: α_i denoting the supply of goods available at supplier O_i $(i \in I)$, β_j the demand at customer $D_j (j \in J)$, p_{ij} the unit transportation cost from supplier $O_i (i \in I)$ to customer D_j $(j \in J)$, η_{ij} the fixed cost for using the route from supplier $O_i (i \in I)$ to customer $D_j (j \in J)$; herein, the parameters such as $\alpha_i, \beta_j, p_{ij}, \eta_{ij} (i \in I, j \in J)$ are non-negative numbers. It is assumed that if a route's transportation amount is positive value (i.e., the route is used), then the route has a fixed cost irrelative to the transportation amount, and a variable cost directly proportional to the quadratic of the transportation amount as a nonlinear term. Under the above-mentioned circumstances, the task is to find such a transportation scheme that the total transportation cost is minimal while satisfying the demand at each customer.

Let x_{ij} be the quantity of goods shipped from supplier O_i $(i \in I)$ to customer D_j $(j \in J)$, which is decision variable to be determined; and let z be the total transportation cost. Introduce zero-one variable y_{ij} such that if $x_{ij} > 0$, then $y_{ij} = 1$, else $y_{ij} = 0$. Then NFCTP can be formulated using the following mixed integer programming model (NFCTP).

(NFCTP):

$$\text{Minimize} \quad z = \sum_{i \in I} \sum_{j \in J} [p_{ij} (x_{ij})^2 + \eta_{ij} y_{ij}]$$

$$\text{subject to} \quad \begin{cases} \sum_{j \in J} x_{ij} \leqslant \alpha_i \quad \forall i \in I; & (1) \\ \sum_{i \in I} x_{ij} \geqslant \beta_j \quad \forall j \in J; & (2) \\ x_{ij} \geqslant 0 \quad \forall i \in I, \quad j \in J; & (3) \\ \text{if } x_{ij} > 0, \quad \text{then } y_{ij} = 1, \quad \text{else } y_{ij} = 0 \quad \forall i \in I, \quad j \in J \end{cases} \quad (4)$$

Following is a typical example, i.e., 3 × 4 problem, to explain more about the presented model (NFCTP). There are three suppliers and four customers. All data are given in Table 1.

In this case, as shown in Fig. 1, the transportation scheme is to ship 10 units from O_1 to D_1, 10 units from O_1 to D_2, 20 units from O_1 to customer D_3, 10 units from O_1 to D_4, 10 units from O_2 to D_2, 10 units from O_2 to D_3, 10 units from O_3 to D_1, 20 units from O_3 to D_2.

Table 1
Data for 3 × 4 problem.

Supplier	Customer								
	Unit transportation cost (p_{ij})				Supply	Fixed cost (η_{ij})			
	D_1	D_2	D_3	D_4		D_1	D_2	D_3	D_4
O_1	8	8	3	5	50	60	88	95	76
O_2	3	5	4	8	20	86	84	70	92
O_3	8	4	5	3	30	67	89	99	89
Demand	20	40	30	10					

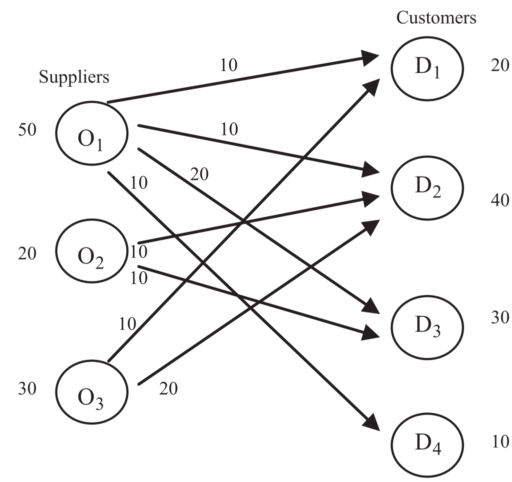

Fig. 1. Transportation scheme graph for 3 × 4 problem.

The total transportation cost of this scheme is as follows:

The fixed cost for shipping from suppliers to customers is $\eta_{11} + \eta_{12} + \eta_{13} + \eta_{14} + \eta_{22} + \eta_{23} + \eta_{31} + \eta_{32} = 60 + 88 + 95 + 76 + 84 + 70 + 67 + 89 = 629$.

The shipping cost from suppliers to customers is

$$\begin{aligned} & p_{11} \times x_{11} \times x_{11} + p_{12} \times x_{12} \times x_{12} + p_{13} \times x_{13} \times x_{13} \\ & \quad + p_{14} \times x_{14} \times x_{14} + p_{22} \times x_{22} \times x_{22} + p_{23} \times x_{23} \times x_{23} \\ & \quad + p_{31} \times x_{31} \times x_{31} + p_{32} \times x_{32} \times x_{32} \\ & = 8 \times 10 \times 10 + 8 \times 10 \times 10 + 3 \times 20 \times 20 + 5 \times 10 \times 10 \\ & \quad + 5 \times 10 \times 10 + 4 \times 10 \times 10 + 8 \times 10 \times 10 \\ & \quad + 4 \times 20 \times 20 = 6600. \end{aligned}$$

The total transportation cost is 629 + 6600 = 7229.

As shown in the typical example, the total transportation cost for model (NFCTP) depends on the quadratic of the shipping units shown as a nonlinear term, which appears extensively in electric power transport systems. Obviously, NFCTP considered by us conforms with that considered by Jo et al. (2007), while it is different from that considered by Hajiaghaei-Keshteli et al. (2010). In fact, what Hajiaghaei-Keshteli et al. (2010) considered is linear FCTP, i.e., commonly called FCTP, which has essentially different objective function from NFCTP (Kannan et al., 2008; Xie and Jia, 2010). It is known that the total transportation cost for the considered NFCTP by us or the commonly called FCTP includes two parts, i.e., the fixed cost, and the variable cost. Because of the fixed charge terms in the objective function, the model for the commonly called FCTP is nonlinear, so is the model for the considered NFCTP by us. However, the variable cost of the used route for the considered NFCTP by us is quadratic function of the shipping units which is nonlinear, while the variable cost of the used route for the commonly called FCTP is linear function of the shipping units. Therefore, for convenience of distinction here, the commonly called FCTP is referred to as linear FCTP.