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Statistical tolerance analysis of a hyperstatic mechanism, using system reliability methods ☆

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ABSTRACT

The quality level of a mechanism can be evaluated *a posteriori* after several months by following the number of warranty returns. However, it is more interesting to evaluate a predicted quality level in the design stage: this is one of the aims of statistical tolerance analysis. A possible method consists of computing the defect probability (P_D) expressed in ppm. It represents the probability that a functional requirement will not be satisfied in mass production. For assembly reasons, many hyperstatic mechanisms require gaps, which their functional requirements depend on. The defect probability assessment of such mechanisms is not straightforward, and requires advanced numerical methods. This problem particularly interests the VALEO W.S. company, which experiences problems with an assembly containing gaps. This paper proposes an innovative methodology to formulate and compute the defect probability of hyperstatic mechanisms with gaps in two steps. First, a complex feasibility problem is converted into a simpler problem. Then the defect probability is efficiently computed thanks to system reliability methods and the m-dimensional multivariate normal distribution Φ_m. Finally, a sensitivity analysis is provided to improve the original design. The whole approach is illustrated with an industrial case study, but can be adapted to other similar problems.

1. Introduction

In very competitive industrial fields such as the automotive industry, more and more interest is being paid to the quality level of manufactured mechanisms. It is very important to avoid warranty returns and manage the rate of out-of-tolerance products in production that can lead to assembly line stoppages and/or the discarding of out-of-tolerance mechanisms.

The quality level of a mechanism can be evaluated by the number of faulty parts in production or by the number of warranty returns per year. However, these two methods of product quality evaluation remain *a posteriori*. Tolerance analysis is a more interesting way to evaluate a predicted quality level in the design stage. Scholtz (1995) proposes a detailed review of classical methods whose goal is to predict functional characteristic variations based on component tolerances. Moreover, statistical tolerance analysis enables the definition of the probability that this functional requirement will be respected, as the well known RSS (Root Sum of Squares) does. Advanced statistical tolerance analysis methods allow the defect probability of an existing design to be computed, knowing the dimension tolerances and functional requirements. Various assumptions about the statistical distributions of component dimensions can be made based on their tolerances. This defect probability, denoted as P_D in the following, is expressed in ppm (parts per million) and predicts the number of faulty parts per million in mass production. Several authors have proposed well-established methodologies to evaluate this probability for linear (Evans, 1975a) or non-linear analytical expressions (Evans, 1975a; Glancy & Chase, 1999; Hassani, Aifaoui, Benamara, & Samper, 2008; Nigam & Turner, 1995) of functional characteristics.

In many cases, engineers design hyperstatic mechanisms to increase rigidity. For assembly reasons, this kind of mechanism requires functional gaps to remove stresses and fulfill its functions. Often, the functional requirements depend on these gaps. A statistical tolerance analysis of mechanisms containing gaps is not straightforward. In the literature, as Ballu, Plantec, and Mathieu (2008) have noted, hyperstatic mechanisms are rarely studied because of their complexity. Moreover, gaps within mechanisms are often neglected or poorly modeled. Valeo W.S., an automotive company for whom quality management is a top priority, with defect probability goals in ppb (parts per billion), is focused on such a mechanism with functional gaps for which existing methodologies are ineffective or unreliable for several reasons.

This paper proposes an innovative methodology able to compute the defect probability of a hyperstatic mechanism containing

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Nomenclature

n	number of parts
$\mathbf{X}$	vector of part dimensions
X_i	ith part dimension
T_i	X_i target value
t_i	X_i tolerance
LSL_i, USL_i	respectively lower and upper specification limits of X_i
σ_i	standard deviation of X_i
μ_i	mean value of X_i
δ_i	mean shift of X_i, difference between target and mean values: $\delta_i = T_i - \mu_i$
$\delta_i^{(max)}$	maximum permissible mean shift of X_i
$C_{pki}^{(r)}$, $C_{pi}^{(r)}$	X_i capability requirements
C_{pki}, C_{pi}	capability measures of X_i
$C_{pi}^{(max)}$	X_i maximum capability level obtained in optimal manufacturing conditions
$\mathbf{g}$	vector of gaps
g_1, g_2	gaps between parts
f_{c1}, f_{c2}	functional characteristics of the mechanism
s	functional requirement threshold (Permissible tightening)
$G(X_i)$	performance function in physical $\mathbf{X}$ space
U_i	ith part dimension in standard space
$H(U_i)$	performance function in standard $\mathbf{U}$ space
P_D	defect probability of the mechanism
$C_{95\%}$	95% confidence interval of a random result
P_j^*	most probable failure point associated with the jth performance function $G_j(\mathbf{X})$
β_j	reliability index associated with the jth performance function $G_j(\mathbf{X})$
Φ	cumulative density function of the standard Gaussian distribution
χ_n^2	cumulative density function of the chi-squared distribution with n degrees of freedom
Φ_m	cumulative density function of the m-dimensional Gaussian distribution
m	number of performance functions
$[\boldsymbol{\rho}]$	covariance matrix
$\boldsymbol{\alpha}^{(j)}$	direction cosines associated with the jth performance function $G_j(\mathbf{X})$
S_i	X_i sensitivity index

gaps. In the literature focused on this field, either the P_D formulation is not adapted to this case study (Ballu et al., 2008; Wu, Dantan, Etienne, Siadat, & Martin, 2009) or the computation method (Monte Carlo) of the defect probability can be improved (Dantan & Qureshi, 2009). The proposed methodology includes a particular formulation of P_D probability and a computation phase. First, a complex feasibility problem, i.e., the research of the existence of multiple non-negative gaps, is converted into a simpler problem consisting of multiple linear equations. Then P_D is efficiently computed thanks to the m-dimensional multivariate normal distribution Φ_m originally used in a system reliability method, the FORM (First Order Reliability Method) system. Moreover, this methodology can be applied to other similar problems. In addition, a brief sensitivity analysis is performed in order to improve the quality of the system with a very low increase in manufacturing cost.

In the following section, assembly issues regarding the tolerance analysis of hyperstatic mechanisms containing gaps are illustrated with the particular VALEO W.S. case. A mathematical formulation of the defect probability P_D is proposed. Taking into account the complexity of this problem, Section 3 describes three available methods to compute P_D including the FORM system one. Two different dimension models, depicting two part manufacturing scenarios, are also proposed. Section 4 compares the different methods and exposes the results of their industrial application. Based on these results, and on a sensitivity analysis, the mechanism is finally redesigned with a very low extra manufacturing cost. Section 5 concludes the paper and presents perspectives for the future.

2. Hyperstatic mechanisms tolerance analysis for assembly issues

2.1. Assembly of a hyperstatic mechanism

A hyperstatic mechanism is overconstrained. When a part is positioned in space it has six degrees of freedom. It can rotate about the three orthogonal axes and move along each of the three axes. In a mechanism, parts are connected to each other by links which eliminate some of these degrees of freedom. If one or more is eliminated more than once, the parts are overconstrained. This creates stresses, and the mechanism is said to be hyperstatic. This situation appears very often. Most of the time, engineers design such systems to increase rigidity. Sometimes, hyperstatic mechanisms are not desired but endured. These kinds of mechanism often involve assembly problems. For this reason, such mechanisms require functional gaps to remove stresses and fulfill their functions.

These gaps, denoted as $\mathbf{g}$ (vector of gaps) in the following, increase the complexity of statistical analysis. They can neither be directly controlled, nor be considered as random variables. Nevertheless, the gap widths are random variables, although they are not independent and depend upon the independent dimensions variables X_i gathered in the vector $\mathbf{X}$. Dantan and Qureshi (2009) introduce the $\exists$ "it exists" quantifier in order to formulate correctly assembly problems concerning mechanisms with gaps. Thus, to ensure mechanism assemblability, at least one feasible gap configuration must be found. The generic defect probability formulation of such a mechanism is:

$$P_D = \text{Prob}\left(\mathbf{X}|\exists \mathbf{g} \in [0; \mathbf{g}_{\max}(\mathbf{X})] : \bigwedge_{i=1}^{m} f_{ci}(\mathbf{X}, \mathbf{g}) \geqslant 0\right)$$

where f_{ci} are functional characteristics which generally have to be positive to ensure assemblability, m is the number of functional requirements and $\mathbf{g}_{\max}$ is the vector of gap widths, depending on $\mathbf{X}$ as mentioned previously. In the interests of simplification, subsequent similar equations are written in the following abbreviated form:

$$P_D = \text{Prob}\left(\exists \mathbf{g} \in [0; \mathbf{g}_{\max}(\mathbf{X})] : \bigwedge_{i=1}^{m} f_{ci}(\mathbf{X}, \mathbf{g}) \geqslant 0\right)$$

As soon as a gap is involved in a functional characteristic, the problem becomes complex. Two different methods can be used to find a feasible gap configuration. It is possible to consider only extreme gap configurations, as Ballu et al. (2008) and Wu et al. (2009) have done, but this method can neglect certain intermediate situations which play a significant role. To be sure to not