Computers & Industrial Engineering 63 (2012) 1178–1188

Contents lists available at SciVerse ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Managing a service system with social interactions: Stability and chaos

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ARTICLE INFO

Article history:
Received 17 May 2011
Received in revised form 22 June 2012
Accepted 25 June 2012
Available online 7 September 2012

Keywords:
Service system
Queueing system
Chaos
Social interaction

ABSTRACT

This paper investigates the dynamic behavior of a service system in terms of the arrival rate in the steady state under the influence of social interactions. Customers are backward looking and rational when making purchasing decisions. Existing customers' re-purchasing decisions are based on their experienced utility – a function of the average waiting time and their expected utility. Potential customers are attracted through social interactions with existing customers. It is shown that the arrival rate of the system in the steady state can exhibit stability, periodic cycles, or chaos due to the effect of social interactions and customers' purchasing behavior. Two examples based on an M/M/1 queueing system illustrate the role of social interactions and the effect of service rates on the stability of the arrival rate in the steady state. The result highlights the dynamical complexity of a simple service system under the impact of customers' behavioral factors, or social interactions. It suggests a new perspective to managing service operations whereby social interactions may play a critical role in the fluctuations of demand.

1. Introduction

There are many factors impacting customers' purchasing behavior in a service system that is repeatedly adopted. The demand for the service is reflected in the fluctuations of the queue of the system and can be measured by the arrival rate. For the managers of the system, fluctuations in arrivals may pose difficulties in capacity and resource management. For the customers of the system, purchasing decisions are not only dependent on the factors inherent in the service, such as the price and the quality, but also on the factors such as the *past purchasing experience*, *the rational expectation*, and the influence of other customers through *social interactions*.

Customers' past purchasing experience and expectation jointly and directly determine if the service is worthy and likely to be *re-purchased*. On the other hand, social interactions indirectly influence a customer's purchasing decision. This is especially true for a new customer whose purchasing decision has no prior experience for reference. The above-mentioned behavioral factors can potentially render the arrival process of a service system into a *complex dynamical system*. It is so even if the service system is run under a deterministic setting, e.g., a constant service rate, price or fixed service quality. The dynamical nature of the arrival process complicates service operations management. The system may turn chaotic. Thus, it is of value to investigate the stability of the arrival process as well as the impact of the underlying behavioral factors in a service system. *Chaos theory* is appropriate for the purpose of this study.

Pioneered by Becker (1974), social interactions are defined as *"particular forms of externalities, in which the actions of a reference group act on an individual's preferences"* (Scheinkman, 2004). The impact of social interactions on customers' purchasing behavior has been widely studied in marketing (Bearden & Etzel, 1982) and economics (Becker, 1991), while few studies have investigated the role of social interactions in the dynamic behavior of a service system. The purpose of this study is to shed light on the impact of customers' behavior and purchasing decisions on the stability of the arrival dynamics in the *steady state* (in the following section, we use *steady state* and *equilibrium* interchangeably). Of particular focus is the effect of social interactions among customers. Based on the dynamical system theory, we analyze the impact of these behavioral factors from a chaos perspective. Specifically, we consider the customers who are *backward looking* (which will be discussed in detail in the following section) when making re-purchasing decisions, and potential customers are attracted due to the social interactions with existing customers. We demonstrate that the dynamics of arrivals in the steady state can be stable, periodic or chaotic due to customer's purchasing behavior under the influence of social interactions.

The structure of the paper is organized as follows. Section 2 briefly reviews relevant literature. Section 3 discusses the model of the service system with heterogeneous backward looking customers in the context of social interactions. The analysis of the model in terms of stability, periodicity and chaos is also provided. Section 4 illustrates the impact of social interactions or the service rate on the dynamic behavior of the service system based on an

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http://dx.doi.org/10.1016/j.cie.2012.06.022

M/M/1 queueing model. Section 5 concludes with a summary, managerial insight, and future research directions.

2. Literature review

The current research is inspired and motivated by two streams of literature. The first stream is about the stability and chaos of a service system (or queueing system). The second stream is about the *equilibrium dynamics* of a complex system in the supply and demand framework. In the following section, we briefly review several key papers within each stream.

Plenty of studies have focused on the stability of dynamical systems. In particular, stability in service systems analyzed from a chaos perspective is an area of great interest. Chase, Serrano, and Ramadge (1993) analyzed the dynamics of the switching time in two discrete control models of a continuous system under full capacity, i.e., a switched arrival system and a switched server system. Their study shows that the switching time in the switched arrival system under a threshold type policy can be chaotic, while the switched server system is generically periodic. Whitt (1993) demonstrated that the dynamic behavior of the queue length at various nodes in a deterministic multiclass network of queues can be chaotic. Feichtinger, Hommes, and Herold (1994) considered a simple discrete time deterministic queueing model with one server and two queues. In their model, the total arrival rate is constant and equal to the service rate; the server adopts a nonlinear decision rule to allocate the service time in each queue based on the difference of the queue length. They discovered that chaotic dynamics appear in the queue length. Haxholdt, Larsen, and van Ackere (2003) demonstrated that in a simple deterministic queueing system, due to the feedback of the customer and the server, the queue length can exhibit the phenomena of sustained oscillation, mode locking, quasi-periodic behavior, or chaos.

Stability or chaos of a service system discussed in the above literature is generally analyzed from the server's perspective, i.e., the supply; few studies have analyzed the impact of various behavioral factors from the customer's perspective, i.e., the demand. Nakayama and Nakamura (2004) is one of the early works in this area. They developed a logit model to study the adoption rate of a fashion where a customer's adoption decision is a discrete choice influenced by social interactions, e.g., the bandwagon effect and the snob effect. Their simulation study shows that, due to social interactions, the adoption rate can be periodic or chaotic under certain conditions.

Rump and Stidham (1998) studied the stability and chaos of the price and the arrival rate in equilibrium in a multiperiod service system. Their result shows that a customer's adaptive expectation can contribute to chaos in the price or the arrival rate dynamics in equilibrium. From economics perspectives, the stability and chaos in a supply and demand system is of great interest. Hommes (1994) investigated the price-quantity dynamics of the cobweb model with adaptive expectations and nonlinear supply and demand curves. He showed that chaotic price dynamics can occur generically, even if both the supply and demand curves are monotonic. Brock and Hommes (1997) focused on the dynamical behavior of the price at the equilibrium in a cobweb type demand–supply model with heterogeneous beliefs of suppliers. They concluded that due to the high intensity of choice to switch predictors, an irregular equilibrium price path can converge to a strange attractor and lead to chaos. Brock, Dindo, and Hommes (2006) generalized the model with forward looking suppliers and showed that forward looking behavior dampens the amplitude of price fluctuation, but local instability of the steady state remains. It is demonstrated that the dynamics of a cobweb type demand–supply model are due to the adaptive, rational or forward looking behavior of suppliers.

The above literature provides the theoretical basis for our study.

3. The model

3.1. Model specification

The service system under consideration is one that provides services that can be repeatedly purchased. To focus on the impact of customers' purchasing behavior on the dynamic characteristics of the service system, we consider the system operating with a constant price and a given quality level. The service is purchased by *heterogeneous customers* whose primary concern is the waiting time in the system (queue and service). In other words, customers consider the waiting time as their major deciding factor before entering the service system. Suppose the queue length is not observable and customers have to estimate the waiting time before purchasing (For example, it may depend on their past experiences). This is actually observed in certain service systems. For example, in a call center, customers cannot observe the queue and the waiting time is the major factor affecting customers' decision to adopt the service or not. The *congestion-prone* systems (Agnew, 1976), such as highways also belong to this category of service systems. Highway users cannot observe the congestion level (or the queue length) of a certain section of the highway, thus they need to estimate the congestion before entering the highway system. (Note: Of course, with modern technology, the congestion information now can be readily available to highway users.) In this paper, we focus on the dynamic behavior of the arrivals in the service system in the steady state, or equilibrium dynamics (Brock et al., 2006) of the service system. We realize that the steady state in real systems may not exist (except for a short time span, since within a short time span, the arrival rate and service rate may not change). Here, we assume the steady state is an approximation or limiting status of a queueing system and the time span of each period is either long enough for the system to converge to the steady state or short enough for a constant arrival and service rate to be observed. Since the queue is not observable, we assume customers adopt the average waiting time in the steady state as their estimation of the time spent in the system. The settings in this paper are similar to those of Rump and Stidham (1998), in which the delay cost is determined by the average waiting time in equilibrium.

Based on the above settings, we assume that customers belong to two groups in each period, either *existing customers* who have purchased the service in the previous period, or *potential customers* who have not. Customers gain *experienced utility* after purchasing the service which only depends on the average waiting time in the steady state. Due to heterogeneity, customers have different *expected utilities* before purchasing. Customers are backward looking, that is the re-purchasing decision of existing customers is based on the comparison between their experienced utility after purchasing and expected utility before purchasing. Potential customers' purchasing decision is solely influenced through social interactions with existing customers. The social interaction effect functions as such that existing customers who are satisfied with the service system will help attract more potential customers (*positive* effect thereafter), while those who are not satisfied will discourage potential customers from purchasing the service (*negative* effect thereafter). As a side note, in the transient stage of a queue, potential customers may be scared away due to a long queue if they care more about the waiting time. However, under the influence of social interactions, more and more customers may be attracted to join the long queue, instead of the short one. Becker (1991) observes this phenomenon where the queue of a seafood restaurant is long while an adjacent seafood restaurant with similar price and quality still has many empty seats available.

For ease of reference, the notations of the key parameters and functions used in this paper are listed as follows: