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A bivariate optimal imperfect preventive maintenance policy for a used system with two-type shocks ☆

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ABSTRACT

This article studies an optimal imperfect preventive maintenance policy based on a cumulative damage model for a used system with initial variable damage. The used system is subject to shocks occurring to a non-homogeneous Poisson process, and suffers one of two types of shocks with stochastic probability: type-I shock (minor) yields a random amount of additive damage of the system, or type-II shock (catastrophic) causes the system to fail. A bivariate preventive maintenance schedule (n,T) is presented in which the system undergoes preventive maintenance at a planned time T and the nth type-I shock, or corrective maintenance at any type-II shock and the total damage exceeds a threshold level, whichever occurs first. The optimal preventive maintenance schedule which minimizes the expected cost rate is derived analytically and discussed numerically.

1. Introduction

Maintenance is carried out to retain a system in or restore it to an acceptable operating condition and can be classified by two major categories: corrective maintenance (CM) or preventive maintenance (PM). CM is the action that occurs when a system fails and some researchers refer to CM as repair, whereas PM is the action taken on a system while it is still operating. It is of great importance to avoid the failure of a system during actual operation when such an event is costly and/or dangerous. The PM policy (schedule) is adapted to slow the degradation process of the operating system and to extend the system life. Determining an optimal PM policy is a major component of reliability theory.

The effects of a maintenance activity are usually described by different restoration degrees (minimal, imperfect, perfect) of the maintained system. A more realistic assumption is that the system after maintenance (either PM or CM) lies somewhere between as good as new and as bad as immediately before the maintenance action. Applying this assumption to maintenance is known as the imperfect maintenance model (Ahmad & Kamaruddin, 2012; Doyen & Gaudoin, 2011; Pham & Wang, 1996). One way of modeling an imperfect maintenance activity is to assume that a perfect maintenance/repair occurs with probability p and a minimal repair occurs with probability $1 - p$, independently of the previous history of repair and maintenance (Brown & Proschan, 1983). Block, Broges, and ans Savits (1985) generalized the Brown–Proschan model (1983) by allowing the probability of a perfect repair to depend on the age of the failed system. Other approaches are to model imperfect maintenance effects directly such as the virtual age model (Kijima, 1989; Kijima, Morimura, & Sujuki, 1988; Sheu, Li, & Chang, 2012), the improvement factor method (Nakagawa, 1988; Sheu, Chang, & Chen, 2012), or changing failure times (Nakagawa, 2005; Pham & Wang, 2003). Some recent applications of the imperfect maintenance model can be found in Liu and Huang (2010), Tsaia, Liua, and Lio (2011) and Lu, Chen, and Liu (2012). In this article, an imperfect maintenance model is considered that the accumulated damage of a used system will reduce to an initial damage due to any maintenance action.

In some real situations, it may be more economical to work a used system than do a new one in the case where the cost of maintenance is much less than the one of replacement (Qian, Ito, & Nakagawa, 2005; Yeh, Lo, & Yu, 2011). Some optimal replacement policies create the used systems of age varying randomly (Murthy & Nguyen, 1982; Sheu & Griffith, 2002). The optimal replacement policies for a used system with initial damage have been studied in Nakagawa (1979), Qian et al. (2005) and Zhao, Nakagawa, and Qian (2012). However, an initial damage of the used system at time 0 since the last maintenance may be a random variable instead of a constant.

Consider a system subject to shocks that weaken the system and make it more expensive to run. A variety of optimal replacement policies subject to shocks have been studied extensively by Sheu, Chang, and Chien (2011), Chang, Sheu, Chen, and Zhang (2011) and Sheu, Chang, Zhang, and Chien (2012). If a system

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Nomenclature

$\{N(t):t \geqslant 0\}$	non-homogeneous Poisson process (NHPP) with intensity $\lambda(t)$
$\Lambda(t)$	mean value function of $\{N(t):t \geqslant 0\}$; $\Lambda(t)=\int_0^t \lambda(u)\mathrm{d}u$
W_0	initial damage of the system at time 0
K	total damage limit of the system
$G_0(x)$	cumulative distribution function (CDF) of W_0 for $x \leqslant K$
p,q	probabilities of type-I shock and type-II shock, respectively; $p+q=1$
W_j	amount of damage due to the jth type-I shock for $j=1,2,3,\ldots$
$G(x)$	CDF of W_j for $x \leqslant K$
Z_j	total damage until the jth type-I shock for $j=1,2,3,\ldots$
$G^{(j)}(x)$	CDF of $W_1+W_2+\cdots+W_j$
$F_{Z_j}(z)$	CDF of Z_j
$\{N_1(t):t \geqslant 0\}$	NHPP with intensity $q\lambda(t)$
$\{N_2(t):t \geqslant 0\}$	NHPP with intensity $p\lambda(t)$
T	planned PM time
n	scheduled number of type-I shocks
Y	life time between successive type-II shocks when n, T, K are infinite
$\overline{F}_p(t)$	CDF of Y
$p_j(t)$	probability of j type-I shocks occurrence
S_j	arrival instant of the jth type-I shock
$F_{S_j}(t), f_{S_j}(t)$	CDF and probability density function (PDF) of S_j, respectively
P_T, P_n, P_{II}, P_K	probabilities of State 1–4, respectively
C_1, C_2	costs of PM and CM, respectively
$c(z)$	maintenance cost of damage z
(n,T)	PM policy based on the age T or the number of type-I shocks
$J(n,T)$	expected cost rate under policy (n,T)
U,V	length and operating cost of a maintenance cycle, respectively
T^*	optimal T that minimizes the expected cost rate
n^*	optimal n that minimizes the expected cost rate
$(n,T)^*$	joint optimal (n,T) that minimizes the expected cost rate
K^*	optimal K that minimizes the expected cost rate

suffers damage due to shocks and fails when the total amount of damage exceeds a threshold level, such a stochastic model generates a cumulative process (Cox, 1962). Some maintenance or replacement policies for a cumulative damage model were studied in Nakagawa and Kijima (1989), Qian et al. (2005). Applying a maintenance policy to a cumulative damage is worth discussing in reliability theory. As the crack growth model for aircrafts (Scarf, Wang, & Laycock, 1996; Sobczyk & Trebicki, 1989), it has been well-known that the unit fails when the total sizes of all cracks attain to a certain level. However, the damage model could be also applied to the backup policies of database systems by replacing *shock* by *update* and *damage* by *dumped file* (Qian, Nakamura, & Nakagawa, 1999), and the garbage collection policies of memory management by replacing *shock* by *collection* and *damage* by *copied objects* (Zhao, Nakamura, & Nakagawa, 2011).

In this article, an optimal imperfect preventive maintenance policy for a used system subject to random shocks is considered. The used system with initial variable damage begins to operate at time 0 since the last maintenance. Suppose that the used system suffers either minor shocks or catastrophic shocks with stochastic probability. Each minor shock causes a random amount of damage of the system, and each damage is additive. The system undergoes preventive maintenance at a planned time T and at the nth minor shock, or it experiences a failure followed by corrective maintenance at any catastrophic shock and at total amount of accumulative damage exceeding a threshold level K.

This article presents a bivariate maintenance policy (n,T) combined with control rule K-based maintenance strategy for a used system subject to random shocks. The rest of this article is organized as follows. Section 2 presents the model formulation and develops the expected cost rate functions. Section 3 focuses on the optimization of the maintenance policy. Section 4 develops some algorithms for determining the optimal PM schedule $(n,T)^*$ and the optimal cumulative damage limit K^*, and some computational examples are given to illustrate the application of algorithms. Finally, Section 5 concludes.

2. Assumptions

(1) PM and CM are completed instantaneously and shocks are detected immediately.
(2) After a maintenance (PM or CM), the system with initial variable damage begins to work at time 0, and the shock process is reset to 0.
(3) A maintenance cycle is defined as the time interval between two consecutive maintenances.
(4) The survival function $\overline{\Phi}(x) \equiv 1-\Phi(x)$ for any distribution function $\Phi(x)$.
(5) The costs for maintenance are ordered $C_2 \geqslant C_1$, which means that a CM cost is greater than a PM cost.

3. System description and model formulation

3.1. General model

We structure a maintenance model for a used system subject to shocks with PM or CM according to the following scheme:

(1) It is assumed that a system with initial damage W_0 begins to work at time 0, where W_0 is a random variable with distribution function $G_0(x) \equiv P(W_0 \leqslant x)$ for $x \leqslant K$ and finite mean μ_0. K is a pre-determined damage limit for an operating system, and the system is in unacceptable (or failure) state when the total damage exceeds K. The system is regarded as a used system.
(2) The system is subject to shocks that arrive according to a non-homogeneous Poisson process (NHPP) $\{N(t):t \geqslant 0\}$ with intensity function $\lambda(t)$ and mean-value function $\Lambda(t)$.
(3) Shocks can be divided into two types. One is type-I shock (minor) with probability q at which it suffers a random amount of additive damage of the system. The other is type-II shock (catastrophic) with probability $p(=1-q)$ at which the system fails. Suppose that an amount W_j of damage due to the jth type-I shock has a distribution function $G(x)=P\{W_j \leqslant x\}$ for $j=1,2,\ldots$
(4) The total damage $Z_j=\sum_{i=0}^{j} W_i$ $(j=1,2,\ldots)$ up to the jth type-I shock has a distribution function

$$F_{Z_j}(z) \equiv P(Z_j \leqslant z)=\int_0^z G^{(j)}(z-x)\mathrm{d}G_0(x), \quad (1)$$

where $Z_0 \equiv W_0$, $G^{(j)}(x)$ is the j-fold Stieltjes convolution of $G(x)$ with itself, and $G^{(0)}(x) \equiv 1$.