Computers & Industrial Engineering 63 (2012) 1235–1242

Contents lists available at SciVerse ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Revised multi-segment goal programming: Percentage goal programming

Ching-Ter Chang, Huang-Mu Chen, Zheng-Yun Zhuang *

The Graduate Institute of Business Management, Chang Gung University, (333) 259 Wen-Hwa 1st Road, Kwei-Shan, Tao-Yuan, Taiwan

ARTICLE INFO

Article history:
Received 16 February 2012
Received in revised form 24 August 2012
Accepted 27 August 2012
Available online 12 September 2012

Keywords:
Goal programming
Multi-segment
Multi-choice
Percentage
Coefficient
In-between selection

ABSTRACT

The multi-segment goal programming (MSGP) model is an extension model of GP wherein the core thinking is inherited from the multi-choice goal programming (MCGP) model. In this paper, we recommend certain points of the MSGP model and offer a Revised MSGP Model as an aid to burdened decision makers who cannot expect an either-or selection of coefficients in practice. The proposed model takes into account a scenario in which the selection of all possible coefficients pertaining to each decision variable in the MSGP model can be an in-between selection instead of an exclusive-or selection. We hope this study can fill in a possible gap that might exist when applying the MSGP model, and can offer an extension model for practitioners when they use this model to solve related decision problems.

1. Introduction

Goal programming (GP) has gained wide popularity since it was proposed by Charnes and Cooper (1961). Many extension models with descendants have been derived in the past decades (Aouni & Kettani, 2001). These include weighted GP, fuzzy GP, lexicographical GP, min–max GP, integer GP and multi-choice GP (MCGP) (Chang, 2007; Lin, 2004; Romero, 2001; Tamiz, Jones, & El-Darzi, 1995; Tamiz, Jones, & Romero, 1998; Zimmermann, 1978). Other decision-making process-related or decision maker (DM)'s preference incorporated models have also been proposed, such as interactive GP (Dyer, 1972), the promethee method with GP (Martel & Aouni, 1990) and GP with utility functions (Chang, 2011). Also, many integrated GP models such as the Fuzzy MCGP (Bankian-Tabrizi, Shahanaghi, & Jabalameli, 2012) have been proposed, while some GP mathematicians have become passionate about the equivalence construal of the models (Mohamed, 1997).

Among the models mentioned above, the MCGP model (Chang, 2007, 2008) is a GP model that allows the right-hand-side (RHS) of each goal to be varied among two or more aspiration levels. With MCGP, a DM can consider multiple levels of aspired target values for each goal. This model is particularly helpful for DMs who are not that certain about their own aspired levels of goals, and is especially helpful for the internal control of enterprise general managers who would not like to see their sales potential under-estimated or cost over-estimated. Following the core spirit of MCGP, Multi-segment GP (MSGP) has been proposed by Liao (2009). In the MSGP model, the coefficient on the left-hand-side (LHS) allows the DMs to set multiple segments of a coefficient on the LHS for decision variables.

As Liao (2009) has mentioned, "if only two-segment aspiration levels exist in each market, this is a case of a multi-objective decision making (MODM) problem with an either-selection". In real cases of pricing issues in marketing, there is a common problem faced by DMs, especially when price discrimination (PD) policy is executed. Consider a simple case in which we would like to sell a product to two groups of customers, VIP customers (VIPC) and normal customers (NC). In this case, the product could be marked with two different prices. The MSGP model is then very suitable for being applied to a vendor who sells many types of product to either VIPC or NC at the same time. However, as Liao (2009) has mentioned, the model is an 'either-selection'. This implies that the capability of the MSGP model is restricted to the PD problem, due to the fact that the uncertain but possible coefficients pertaining to each decision variable in the MSGP model have an exclusive-or relationship.

To show this problem, take the following example from Liao (2009),

(P1)

(Goals)

$$g_1:\ (3 \text{ or } 6)x_1 + 2x_2 + x_3 = 115 \quad (1)$$

$$g_2:\ 4x_1 + (5 \text{ or } 9)x_2 + 2x_3 = 80 \quad (2)$$

$$g_3:\ 3.5x_1 + 5x_2 + (7 \text{ or } 10)x_3 = 110 \quad (3)$$

(Constraints)

$$x_2 + x_3 \geqslant 9,\ x_2 \geqslant 5,\ x_1 + x_2 + x_3 \geqslant 21 \quad (4)$$

* Corresponding author. Tel.: +886 3 2118800x3713/3694; fax: +886 3 2118212.
E-mail addresses: chingter@mail.cgu.edu.tw (C.-T. Chang), jckychen@gmail.com (H.-M. Chen), waynemcgwire@yahoo.com, d9940001@stmail.cgu.edu.tw (Z.-Y. Zhuang).

0360-8352/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.cie.2012.08.005

P1 can be expressed by the MSGP model as follows:

$$
\begin{aligned}
\min \quad & z = d_1^+ + d_1^- + d_2^+ + d_2^- + d_3^+ + d_3^- \\
\text{s.t.} \quad & (3b_1 + 6(1-b_1))x_1 + 2x_2 + x_3 - d_1^+ + d_1^- = 115 \\
& 4x_1 + (5b_2 + 9(1-b_2))x_2 + 2x_3 - d_2^+ + d_2^{=} = 80 \\
& 3.5x_1 + 5x_2 + (7b_3 + 10(1-b_3))x_3 - d_3^+ + d_3^- = 110 \\
& d_i^+,\ d_i^- \geqslant 0,\ i = 1,2,3
\end{aligned}
$$

where b_1, b_2, and b_3 are binary variables; d_i^+ and d_i^- are the positive and negative deviation variables, respectively.

The solution obtained by Liao (2009) is as follows:

$$(x_1, x_2, x_3, b_1, b_2, b_3) = (11.5\bar{4}, 5, 4, 4\bar{6}, 0, 1, 0)$$

As the result, for g_1, the coefficient 6 for decision variable x_1 takes effect, while the coefficient 5 for decision variable x_2 takes effect for g_2, and the coefficient 10 for x_3 takes effect for g_3.

This study proposes an extension model of MSGP, called the Revised MSGP Model (RMSGP), wherein all possible coefficients of each decision variable, x_j, are not alone. In order to deal with this concern, a concept of mixture percentage (τ_j) of the possible coefficients is introduced. With P1 for example, such a concept represents the percentage of goods to be sold to one customer group, as opposed to another. That is, taking the two uncertain coefficients (i.e., 3 or 6) for x_1 in g_1 of P1, the proposed model can determine a mixture percentage, τ_1^* (the solution for the variable τ_j) that represents how the two coefficients act together when being solved, and $\tau_1^* x_1^*$ and $(1-\tau_1^*)x_1^*$ indicate the exact quantities to be sold to the VIPC and NC, respectively. This is especially useful for marketing decisions, where it is necessary to decide a proper product portfolio in advance.

A percentage GP approach (%GP) is also proposed to solve problems modeled using the RMSGP. This approach is capable of solving an uncertain decision variable coefficient that is continuous between both ends. In fact, uncertainties of objective function coefficients, of constraint's RHS values and of constraint's LHS coefficients, are the three main categories of uncertainties in linear programming (LP) models. These uncertainties can be represented by interval numbers or fuzzy numbers (Tong, 1994). The uncertain coefficient modeled by %GP is somehow akin to the interval number concept of the uncertain constraint's LHS coefficients category. Nevertheless, most existing approaches solve problems with coefficients that have intrinsic uncertainty. While the application scenario of the MSGP/RMSGP is not intrinsically uncertain, it requires the solution of a "non-intrinsic, in-between uncertainty", wherein the two ends of the interval are known and fixed prior to modeling. This raises the need for a %GP approach that appropriates the scenario of the RMSGP.

In summary, by furnishing the possible flaw of MSGP, this study not only provides supplements to the trends of MCGP and MSGP research, but also improves to the field of GP models. The practical use of the proposed model also can be expected.

The remainder of this paper is organized as follows: Section 2 describes the details from the prototype of MSGP to the proposed RMSGP model as well as the %GP approach. A brief example with contextual settings is also given in the section to show how the proposed model can assist the decision making process in marketing considering product portfolio and PD. By taking an exemplar from the previous MSGP study, Section 3 demonstrates the solution process of RMSGP by %GP and compares the result against that of traditional MSGP (solved by mixed binary GP). Section 4 encompasses the implications and contributions of the proposed model, and Section 5 offers concluding.

2. Modeling the percentage GP

2.1. The Revised MSGP Model

Akin to any GP model, the simultaneous equations of the MSGP model can be represented in a matrix form as follows:

$$
\begin{aligned}
&(\text{MSGP}) \\
&\min\ (+)\ (D^+ + D^-) \\
&CX - D^+ + D^- = A
\end{aligned}
$$

where X is the $n \times 1$ decision vector, C is a $k \times n$ matrix of coefficients wherein each element c_{ij} contains possible coefficients semantically linked with the exclusive-or (XOR) operator; D^+ and D^- are $n \times 1$ vectors of surplus and slack deviations; (+) is the direct sum operator taking all the elements of the $n \times 1$ vector as summands, and A is the $k \times 1$ vector that describes the aspiration levels of each of the k goals.

Thus, P1 from the MSGP study (Liao, 2009) can be written as follows:

$$
\begin{aligned}
&(\text{P2}) \\
&(\text{Goal}) \\
&\begin{bmatrix} (3\ \text{XOR}\ 6) & 2 & 1 \\ 4 & (5\ \text{XOR}\ 9) & 2 \\ \frac{7}{2} & 5 & (7\ \text{XOR}\ 10) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 115 \\ 80 \\ 110 \end{bmatrix}, \text{ with constraint } (4)
\end{aligned}
$$

For the determination of an appropriate coefficient value in-between an interval (e.g. in-between 3 and 6), coefficients transformation from discrete numbers to interval numbers is needed (we adhere to the interval coefficient expression in Tong's research (1994)):

$$
\begin{aligned}
&(\text{P3}) \\
&\begin{bmatrix} [3,6] & 2 & 1 \\ 4 & [5,9] & 2 \\ \frac{7}{2} & 5 & [7,10] \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 115 \\ 80 \\ 110 \end{bmatrix}, \text{ with constraint } (4)
\end{aligned}
$$

These simultaneous equations can be re-written as follows:

$$
\begin{aligned}
&(\text{P4}) \\
&\begin{bmatrix} \xi_{11}(3,6) & \xi_{12}(2,2) & \xi_{13}(1,1) \\ \xi_{21}(4,4) & \xi_{22}(5,9) & \xi_{23}(2,2) \\ \xi_{31}(\frac{7}{2},\frac{7}{2}) & \xi_{32}(5,5) & \xi_{33}(7,10) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 115 \\ 80 \\ 110 \end{bmatrix}, \text{ with constraint } (4)
\end{aligned}
$$

where $\xi_{11}(3,6)$ is a "function" in terms of an upper bound of the coefficient 6 and a lower bound of the coefficient 3 (the output value of $\xi_{11}(3,6) \in [3,6]$), $\xi_{12}(2,2)$ is a function that produces a coefficient value of 2; other coefficients in terms of $\xi_{ij}(a_{ij}, b_{ij})$ functions can be recognized correspondingly, in which i and j are indices for the coefficient function of the jth decision variable (x_j) in the ith goal criteria.

Thus, the coefficient matrix C in the MSGP model can be no longer a matrix in which the elements (c_{ij}) are simply or-linked values. It becomes another matrix Θ with function elements $\xi_{ij}(a_{ij}, b_{ij})$, each of which is a 'coefficient function'. Therefore, the model can be re-defined as follows:

$$
\begin{aligned}
&(\text{Revised MSGP}) \\
&\min\ (+)\ (D^+ + D^-) \\
&\Theta X - D^+ + D^- = A
\end{aligned}
$$

where Θ is a $k \times n$ matrix of coefficients wherein each element $\xi_{ij}(a_{ij}, b_{ij})$ is a 'coefficient function' in terms of its upper and lower bounds a_{ij} and b_{ij}; other variables are defined as in (MSGP).