Computers & Industrial Engineering 71 (2014) 27–36

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

An approach to group decision making with heterogeneous incomplete uncertain preference relations ☆

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ARTICLE INFO

Article history:
Received 26 April 2012
Received in revised form 30 December 2013
Accepted 8 February 2014
Available online 26 February 2014

Keywords:
Group decision making
Incomplete uncertain preference relation
Priority weight
Optimization

ABSTRACT

For practical group decision making problems, decision makers tend to provide heterogeneous uncertain preference relations due to the uncertainty of the decision environment and the difference of cultures and education backgrounds. Sometimes, decision makers may not have an in-depth knowledge of the problem to be solved and provide incomplete preference relations. In this paper, we focus on group decision making (GDM) problems with heterogeneous incomplete uncertain preference relations, including uncertain multiplicative preference relations, uncertain fuzzy preference relations, uncertain linguistic preference relations and intuitionistic fuzzy preference relations. To deal with such GDM problems, a decision analysis method is proposed. Based on the multiplicative consistency of uncertain preference relations, a bi-objective optimization model which aims to maximize both the group consensus and the individual consistency of each decision maker is established. By solving the optimization model, the priority weights of alternatives can be obtained. Finally, some illustrative examples are used to show the feasibility and effectiveness of the proposed method.

1. Introduction

The increasing complexity of the socio-economic environment makes it less and less possible for a single decision maker (DM) to consider all relevant aspects of a decision making problem (Kim, Choi, & Kim, 1999). Therefore, many decision making problems in the real world are usually conducted by decision groups, and group decision making (GDM) problem has long been identified as a hot topic in decision science research area (Hwang & Lin, 1987).

For a typical GDM problem, decision makers are usually asked to provide their preferences over a set of alternatives (criteria). As an effective tool, preference relation has been widely used to express decision makers' preference information through pairwise comparisons. Up to now, many formats of preference relations have been developed (Xu, 2007b), such as multiplicative preference relation (Herrera, Herrera-Viedma, & Chiclana, 2001; Saaty, 1980), fuzzy preference relation (Herrera-Viedma, Chiclana, Herrera, & Alonso, 2007; Tanino, 1984) and linguistic preference relation (Herrera, Herrera-Viedma, & Verdegay, 1996; Xu, 2006; Xu, 2008). But due to the uncertainty of decision environment and the lack of decision makers' knowledge, preference relations given by decision makers sometimes are uncertain ones (Liu, Zhang, & Wang, 2012; Xu, 2004b). As a result, many publications have focused on deriving priority weights from uncertain preference relations (Chen & Zhou, 2012; Gong, Li, Zhou, & Yao, 2009; Wang, Yang, & Xu, 2005; Wu, Li, Li, & Duan, 2009; Xu & Chen, 2008a).

For some complex GDM problems defined with high uncertainty, decision makers may be of different culture and education background and may have different levels of knowledge about the decision making problems (Herrera-Viedma, Herrera, & Chiclana, 2002; Palomares, Rodríguez, & Martínez, 2013). On the other hand, decision makers sometimes are distributed in different areas and it may be difficult for them to reach an agreement on which type of preference relations can be used. In such situations, decision makers may tend to express their preference using different formats of preference relations according to their own will. In recent years, group decision making with heterogeneous preference information has received more and more attention (Delgado, Herrera, Herrera-Viedma, & Martínez, 1998; Espinilla, Palomares, Martínez, & Ruan, 2012; Fan, Xiao, & Hu, 2004; Li, Huang, & Chen, 2010; Pérez, Alonso, Cabrerizo, Lu, & Herrera-Viedma, 2011). For instance, Herrera-Viedma et al. (2002) presented a consensus model for multi-person decision making problems with different

☆ This manuscript was processed by Area Editor Imed Kacem.

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http://dx.doi.org/10.1016/j.cie.2014.02.004
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preference structures to help experts change their opinions and obtain a degree of consensus. Herrera, Martínez, and Sánchez (2005) developed an aggregation process to combine different types of preference relations, such as linguistic, numerical and interval-valued information. Fan, Ma, Jiang, Sun, and Ma (2006) established a goal programming model to solve group decision making problems where the preference information on alternatives is represented in multiplicative preference relations and fuzzy preference relations. Wang and Fan (2007) investigated the aggregation of fuzzy preference relations and multiplicative preference relations. In their approach, they presented two optimization aggregation approaches to determine the relative weights of individual fuzzy preference relations so that they can be aggregated into a collective fuzzy preference relation. Dong, Xu, and Yu (2009) proposed a linguistic multi-person decision making model based on linguistic preference relations which can integrate fuzzy preference relations, different types of multiplicative preference relations and multi-granular linguistic preference relations. In order to deal with GDM problems with heterogeneous incomplete preference relations, including multiplicative preference relations, fuzzy preference relations and linguistic preference relations, Fan and Zhang (2010) established a goal programming model to derive the collective evaluation of alternatives. Like Fan and Zhang (2010)'s study, Xu (2011) considered four formats of incomplete preference relations and established a quadratic programming model to obtain the ranking of alternatives. Pérez, Cabrerizo, and Herrera-Viedma (2010) presented a mobile decision support system for dynamic group decision making with fuzzy preference relations, orderings, utility functions and multiplicative preference relations, in which mobile technologies are applied and the set of alternatives can change throughout the process. Pérez, Cabrerizo, and Herrera-Viedma (2011b) also developed a mobile GDM model for changeable decision environments which allows decision makers to express their preferences using heterogeneous preference relations, including fuzzy preference relations and multi-granularity linguistic preference relations. In a recent work, Palomares et al. (2013) proposed a consensus model in which decision makers can express their opinions by using different types of information, capable of dealing with large groups of decision makers, which incorporates the management of the group's attitude towards consensus by means of the proposed Attitude-OWA operator.

From the above analysis, a lot of studies have been conducted to deal with GDM with heterogeneous preference relations and previous studies have significantly advanced the field of GDM. However, most of the research focuses on GDM problems with certain preference relations. There is very little literature addressing GDM problems with heterogeneous uncertain preference relations. On the other hand, for actual GDM problems there may be cases in which decision makers do not have an in-depth knowledge of the problem to be solved. In such cases, decision makers may not put their opinions forward about certain aspects of the problem, and as a result incomplete preference relations may be obtained (Alonso, Herrera-Viedma, Chiclana, & Herrera, 2009; Alonso, Herrera-Viedma, Chiclana, & Herrera, 2010; Herrera-Viedma et al., 2007; Zhang & Guo, 2013). Considering such situations, the main contribution of this paper is to propose a GDM approach to deriving priority weights from heterogeneous incomplete uncertain preference relations, including uncertain multiplicative preference relations, uncertain fuzzy preference relations, uncertain linguistic preference relations and intuitionistic fuzzy preference relations, which can allow decision makers to express their preference information over alternatives more flexibly. For this purpose, this paper first defines the group consensus index and the collective individual consistency index for the four types of incomplete uncertain preference relations under group decision making environment. Afterwards, a bi-objective optimization model, which aims to obtain both the maximum group consensus and collective individual consistency, is proposed to derive the priority weights.

To do so, the rest of this paper is organized as follows. Section 2 presents some concepts and preliminaries related to incomplete uncertain preference relations. In Section 3, we give a description of the group decision making problem with heterogeneous incomplete uncertain preference relations. Section 4 proposed a bi-objective optimization model to address the group decision making problem. In Section 5, we give some illustrative examples to show the feasibility and effectiveness of the proposed method. Section 6 gives a discussion on the advantages and limitations about the proposed approach. Finally, we conclude this paper in Section 7.

2. Preliminaries

In this section, we present some basic concepts and preliminaries related to incomplete uncertain preference relations, including uncertain multiplicative preference relations, uncertain fuzzy preference relations, uncertain linguistic preference relations and intuitionistic fuzzy preference relations.

For the convenience of analysis, let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite set of alternatives, where x_i denotes the ith alternative, $i \in \{1, 2, \ldots, n\} = N$. In addition, we denote the priority weight vector obtained from a preference relation by $w = (w_1, w_2, \ldots, w_n)^{\mathrm{T}}$, such that $\sum_{i=1}^{n} w_i = 1,\ w_i \geqslant 0,\ i \in N$.

Definition 1 (*Saaty and Vargas, 1987*). A matrix $\tilde{A} = (\tilde{a}_{ij})_{n\times n}$ is called an uncertain multiplicative preference relation if $\tilde{a}_{ij}$ satisfies $\tilde{a}_{ij} = [a_{ij}^-, a_{ij}^+],\ a_{ij}^+ \geqslant a_{ij}^-,\ a_{ij}^- a_{ji}^+ = a_{ij}^+ a_{ji}^- = 1,\ a_{ii}^+ = a_{ii}^- = 1$, where $\tilde{a}_{ij}$ is the interval-valued preference degree to which the alternative x_i is preferred to x_j, and $a_{ij}^-,\ a_{ij}^+ \in \{1/9, 1/8, 1/7, \ldots, 1/2, 1, 2, \ldots, 7, 8, 9\},\ i,\ j \in N$.

Definition 2 (Wang et al., 2005). Let $\tilde{A} = (\tilde{a}_{ij})_{n\times n} = ([a_{ij}^-, a_{ij}^+])_{n\times n}$ be an interval multiplicative preference relation. If there exists a positive vector $w = (w_1, w_2, \ldots, w_n)^{\mathrm{T}}$ such that the following convex feasible region

$$\Theta = \left\{ w = (w_1, w_2, \ldots, w_n)^{\mathrm{T}} | a_{ij}^- \leqslant \frac{w_i}{w_j} \leqslant a_{ij}^+,\ w_i > 0, i, j \in N, \sum_{i=1}^{n} w_i = 1 \right\} \tag{2.1}$$

is nonempty, then $\tilde{A}$ is called a consistent interval multiplicative preference relation.

Definition 3 (Xu, 2004b). A matrix $\tilde{B} = (\tilde{b}_{ij})_{n\times n}$ is called an uncertain fuzzy preference relation if $\tilde{b}_{ij}$ satisfies $\tilde{b}_{ij} = [b_{ij}^-, b_{ij}^+]$, $b_{ij}^+ \geqslant b_{ij}^-,\ b_{ij}^- + b_{ji}^+ = b_{ij}^+ + b_{ji}^- = 1,\ b_{ii}^+ = b_{ii}^- = 0.5$, where $\tilde{b}_{ij}$ is the interval-valued preference degree to which the alternative x_i is preferred to x_j, and $b_{ij}^-,\ b_{ij}^+ \in [0, 1],\ i,\ j \in N$.

Definition 4 (Xu and Chen, 2008a). Let $\tilde{B} = (\tilde{b}_{ij})_{n\times n} = ([b_{ij}^-, b_{ij}^+])_{n\times n}$ be an interval fuzzy preference relation. If there exists a positive vector $w = (w_1, w_2, \ldots, w_n)^{\mathrm{T}}$ such that the following convex feasible region

$$\Theta = \left\{ w = (w_1, w_2, \ldots, w_n)^{\mathrm{T}} | b_{ij}^- \leqslant \frac{w_i}{w_i + w_j} \leqslant b_{ij}^+, w_i > 0, i, j \in N, \sum_{i=1}^{n} w_i = 1 \right\}. \tag{2.2}$$

is nonempty, then $\tilde{B}$ is called a consistent interval fuzzy preference relation.