Computers & Industrial Engineering 65 (2013) 577–585

Contents lists available at SciVerse ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

A priori reformulations for joint rolling-horizon scheduling of materials processing and lot-sizing problem

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ARTICLE INFO

Article history:
Received 27 June 2012
Received in revised form 3 April 2013
Accepted 4 April 2013
Available online 12 April 2013

Keywords:
Lot sizing and scheduling
Facility location reformulation
Valid inequalities
Metaheuristics

ABSTRACT

In many production processes, a key material is prepared and then transformed into different final products. The lot sizing decisions concern not only the production of final products, but also that of material preparation in order to take account of their sequence-dependent setup costs and times. The amount of research in recent years indicates the relevance of this problem in various industrial settings. In this paper, facility location reformulation and strengthening constraints are newly applied to a previous lot-sizing model in order to improve solution quality and computing time. Three alternative metaheuristics are used to fix the setup variables, resulting in much improved performance over previous research, especially regarding the use of the metaheuristics for larger instances.

1. Introduction

The lot sizing problem is frequently encountered in industrial production planning. How does a planner decide the lot size of each product produced on one or more machines in each demand period over a planning horizon? This problem has been extensively researched, as discussed in the reviews by Bahl, Ritzman, and Gupta (1987), Drexl and Kimms (1997), Brahimi, Dauzere-Peres, Najid, and Nordli (2006), Karimi, Fatemi Ghomia, and Wilson (2003) and Jans and Degraeve (2007), Jans and Degraeve (2008).

In some industrial sectors, production setup costs and times are sequence-dependent, so that the decisions about the production of lots concern their sequence as well as their size. The two sets of decisions are mutually dependent and so should be modelled and decided simultaneously rather than separately. For example, Araujo, Arenales, and Clark (2007) consider the processing of key materials at an initial stage as well as the products that use the materials. In such a two-stage system, the sequencing decisions at the end stage must jointly consider products that use the same materials, so that material changeovers are minimised at the prior stage. In other words, the sequencing at both stages must be integrated. The model in Araujo et al. (2007) is based on a classical formulation of the lot sizing problem, but its complexity meant that an established industrial-grade optimisation solver was unable to find an optimal solution within acceptable computing time. As a result, the paper developed heuristic solution methods based on *relax-and-fix* within a rolling horizon approach and incorporating metaheuristics.

Since then, many other authors have also researched the integrated lot-sizing and sequencing problem with material preparation at a prior stage in a wide range of industrial settings. Examples include soft drink production (Ferreira, Clark, Alamada-Lobo, & Morabito, 2012; Ferreira, Morabito, & Rangel, 2009, 2010; Toledo, França, Morabito, & Kimms, 2009), animal feed (Clark, Morabito, & Toso, 2010; Toso, Clark, & Morabito, 2009), electrofused grains (Luche, Morabito, & Pureza, 2009), glass bottles (Almada Lobo, Klabjan, Carravilla, & Oliveira, 2007; Almada Lobo, Oliveira, & Carravilla, 2008), foundries (Araujo, Arenales, & Clark, 2008; Camargo, Toledo, & Mattiollo, 2012; Tonaki & Toledo, 2010), yogurt packaging company (Marinelli, Nenni, & Sforza, 2007), pharmaceutical company (Stadtler, 2011) and sand casting operations (Hans & Van de Velde, 2011).

Most of this recent research, including Araujo et al. (2007), is based on the *General Lot Sizing and Scheduling Problem* (GLSP) model (Fleischmann & Meyr, 1997) in which the planning horizon is subdivided into macro-periods, in each of which multiple products can be produced. To model the sequence of lots, each macro-period is in turn subdivided into micro-periods in which at most one product can be produced. This special structure involving subperiods within macro time periods is similar to a small-bucket framework (Koçlar, 2005).

However, some papers, such as Clark et al. (2010) and Ferreira et al. (2012) take a different approach, using an asymmetric travelling salesman problem (ATSP) representation for sequencing lots

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http://dx.doi.org/10.1016/j.cie.2013.04.003

rather than a GLSP-type model. The results presented in Ferreira et al. (2012) shown the superiority of their ATSP-type model over a GLSP-type model. One possible reason is the poor quality of the GLSP linear relaxation as a lower bound on the optimal solution.

Taking forward research initiated in Bernardes, Araujo, and Rangel (2010), the first contribution of this current paper is to demonstrate that certain reformulations applied to the GLSP-type model in Araujo et al. (2007) can provide improved solutions using established optimisation solvers, due mainly to their better quality of linear-relaxation lower-bounds, contributing to the growing research in this area. The second contribution is to show computationally that the use of metaheuristic methods can help solve the reformulations more quickly and better than established solvers, such as, Cplex 12.0.

This paper is structured as follows. In Section 2, the original model in Araujo et al. (2007) is presented. Section 3 develops extended formulations and proposes new constraints. In Section 4, a reformulated rolling horizon-based model is proposed while Section 5 presents the metaheuristics. Section 6 computationally compares the quality of the reformulations using the *Branch-and-Cut* search within the solver Cplex 12.0 and using the metaheuristics. Section 7 concludes and poses challenges for future research.

2. Original Formulation (OF)

In Araujo et al. (2007), a material may be used in multiple products, but a product is made from just one material. A product must be manufactured in the same time period in which its material is processed. Thus processed materials cannot be held over from one period to the next. In each time period only one material can be processed on a given machine. A setup changeover from one material to another is sequence-dependent, i.e., it consumes capacity time in a manner that is dependent on the sequence in which the materials are processed. The triangle inequality holds for setup costs and times so that it is optimal to produce at most one lot per product per period. The model allows backlogs as well as inventory.

The sequencing decisions are made by dividing a period into smaller subperiods, as in the *General Lot Sizing and Scheduling Problem* (GLSP) model (Drexl & Kimms, 1997; Fleischmann & Meyr, 1997; Meyr, 2000, 2002). Let K be the total number of materials, P the total number of products, T the total number of periods and η the total number of subperiods. Consider the following indices and data:

Indices	
	$j, k = 1,\ldots,K$ processed materials
	$p = 1,\ldots,P$ products
	$t = 1,\ldots,T$ periods
	$n = 1,\ldots,\eta$ subperiods
Data	
C	Capacity available on the machine in each subperiod
ρ_p	Capacity required to produce one unit of product p
d_{pt}	Demand for product p at the end of period t
$S(k)$	Set of products p that use material k. Each product uses just one material, i.e., $\{1,\ldots,P\} = S(1) \cup,\ldots,\cup S(K)$, and $S(k) \cap S(j) = \emptyset$, for all materials $k \neq j$, implying $\sum_k \lvert S(k)\rvert = P$
h^-_{pt}	Backlog penalty for delaying delivery of a unit of product p at the end of period t
h^+_{pt}	Inventory penalty for holding a unit of product p at the end of period t
s_{jk}	Setup penalty (or cost) for changing over from material j to material k, where $s_{jj} = 0$
st_{jk}	Setup time (loss of machine capacity) for changing over from material j to material k, where $st_{jj} = 0$
Variables	
x_{pn}	Quantity (lot-size) of product p to be produced in subperiod n
I^+_{pt}	Inventory of product p at the end of period t, where I^+_{p0} is the initial inventory at the start of period 1.
I^-_{pt}	Backlog of product p at the end of period t, where I^-_{p0} is the initial backlog at the start of period 1.
y^k_n	Binary variable: $y^k_n = 1$ if the machine is configured for production of material k in subperiod n, otherwise $y^k_n = 0$. Note that the initial setup state y^k_0 is set to zero and so not a variable
z^{jk}_n	Binary setup variable: $z^{jk}_n = 1$ if there is a machine changeover from material j to material k at the start of subperiod n, otherwise $z^{jk}_n = 0$. Thus $z^{jk}_n = 1$ if $y^j_{n-1} = 1 \& y^k_n = 1$, and $z^{jk}_n = 0$ if $y^j_{n-1} = 0$ or $y^k_n = 0$. It is relaxed to be continuous for reasons explained below
Consider also the following definitions from the GLSP model	
η_t	Maximum number of subperiods in period t
$Ft = 1 + \sum_{\tau=1}^{t-1} \eta_t$	First subperiod in period t
$L_t = F_t + \eta_t - 1$	Last subperiod in period t
$\eta = \sum_{t=1}^{T} \eta_t$	Total number of subperiods over periods $1,\ldots,T$

The *Original Formulation* (OF) (Araujo et al., 2007) of the mathematical model is:

$$\text{Minimise} \quad \sum_p \sum_t (h^-_{pt} I^-_{pt} + h^+_{pt} I^+_{pt}) + \sum_j \sum_k \sum_{n=F_1}^{L_T} s_{jk} z^{jk}_n \tag{1}$$

subject to

$$I^+_{p,t-1} - I^-_{p,t-1} + \sum_{n=F_t}^{L_t} x_{pn} - I^+_{pt} + I^-_{pt} = d_{pt} \quad \forall p, t \tag{2}$$

$$\sum_{p \in S(k)} \rho_p x_{pn} + st_{jk} z^{jk}_n \leqslant C y^k_n \quad \forall j, k, n = F_1, \ldots, L_T \tag{3}$$

$$z^{jk}_n \geqslant y^j_{n-1} + y^k_n - 1 \quad \forall j, k, n = F_1, \ldots, L_T \tag{4}$$

$$\sum_k y^k_n = 1 \quad \forall n = F_1, \ldots, L_T \tag{5}$$