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Single-machine batch delivery scheduling with an assignable common due date and controllable processing times

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ABSTRACT

We consider single-machine batch delivery scheduling with an assignable common due date and controllable processing times, which vary as a convex function of the amounts of a continuously divisible common resource allocated to individual jobs. Finished jobs are delivered in batches and there is no capacity limit on each delivery batch. We first provide an $O(n^5)$ dynamic programming algorithm to find the optimal job sequence, the partition of the job sequence into batches, the assigned common due date, and the resource allocation that minimize a cost function based on earliness, tardiness, job holding, due date assignment, batch delivery, and resource consumption. We show that a special case of the problem can be solved by a lower-order polynomial algorithm. We then study the problem of finding the optimal solution to minimize the total cost of earliness, tardiness, job holding, and due date assignment, subject to limited resource availability, and develop an $O(n\log n)$ algorithm to solve it.

1. Introduction

The importance of just-in-time scheduling has led to a wide range of investigations of scheduling problems that include both earliness and tardiness penalties. In the just-in-time scheduling environment, if too many orders are completed before a specified delivery date (or due date), they must be held in storage, thus incurring holding and/or tying up capital for the company. If the company promises its customers too soon a delivery date, many orders may not be able to be completed before the delivery date due to capacity and resource constraints. This also entails extra costs including late charges and tardiness penalties. In order to reduce earliness–tardiness penalties, including the possibility of losing customers, companies are under increasing pressure to quote attainable delivery dates. At the same time, promising delivery dates too far into the future may turn customers away or may compel companies to offer price discounts in order to retain customers. Thus there is an important tradeoff between assigning relatively short due dates to customer orders and reducing tardiness penalty. Consequently, many recent studies consider due date assignment as part of the scheduling process in view of the fact that the ability to control due dates is a major factor for improving system performance. The most common due date assignment methods used in manufacturing include (i) the common (or constant) due date assignment method (usually referred to as *CON*), where all the jobs are assigned the same due date; (ii) the slack due date assignment method (usually referred to as *SLK*), where the jobs are given an equal flow allowance that reflects equal waiting time (i.e., equal slacks); and (iii) the unrestricted (or different) due date assignment method (usually referred to as *DIF*), where each job can be assigned a different due date with no restrictions. For reviews of research results on scheduling models considering due date assignment and their practical applications, the reader may refer to Cheng and Gupta (1989), Gordon, Proth, and Chu (2002), Gordon, Strusevich, and Dolgui (2010), and Lauff and Werner (2004).

Recently, increasing attention has been paid to due date scheduling problems in which the jobs have controllable (compressible) processing times. The notion of controllable processing times arises from project planning and control. The assumption of controllable processing times is justified in situations where jobs can be accomplished in shorter or longer durations by increasing or decreasing the allocation of resources to process individual jobs. Studies of scheduling problems with controllable processing times were initiated by Vickson (1980a, 1980b). Surveys of this area of scheduling research can be found in Nowicki and Zdrzalka (1990) and Shabtay and Steiner (2007a). In most studies of scheduling with controllable processing times, researchers assume that the job processing time is a bounded linear function of the amount of resources allocated to process the job (e.g., Daniels, 1990; Janiak, 1987; Janiak & Kovalyov, 1996; Kayvanfar, Mahdavi, & Komaki, 2012; Ng, Cheng, & Kovalyov, 2004; Panwalkar & Rajagopalan,

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Nomenclature

n	number of jobs to be processed at time zero ($n \geqslant 2$)
J_j	job number j
$[j]$	a subscript denoting the job assigned in position j of a given sequence
p_j	processing time of job J_j
$\bar{p}_j$	noncompressed processing time of job J_j
u_j	amount of resource allocated to job J_j
$\bar{u}_j$	upper bound on the amount of resource that can be allocated to job J_j
a_j	positive compression rate of job J_j
w_j	workload of the processing operation of job J_j
k	a positive constant
d	common due date for all the jobs, which is a decision variable
C_j	completion time of job J_j
D_j	delivery time of job J_j
$E_j = \max\{0, d - D_j\}$	earliness of job J_j
$T_j = \max\{0, D_j - d\}$	the tardiness of job J_j
$H_j = D_j - C_j$	holding time of job J_j
α	cost of one unit of earliness
β	cost of one unit of tardiness
θ	cost of one unit of holding time
γ	cost of one unit of due date time
ψ	unit batch delivery cost
λ	a Lagrangian multiplier (shadow price)
v_j	cost of allocating one unit of the resource to process job J_j
B_k	set of jobs contained in the kth batch
$\|B_k\|$	number of jobs in B_k
l_k	number of jobs in the first k batches with $l_0 = 0$, where $j = 1,2,\ldots,n$ and $k = 0,1,2,\ldots$

1992; Van Wassenhove & Baker, 1982), i.e., the resource consumption function is of the form $p_j(u_j) = \bar{p}_j - a_j u_j,\ j = 1,2,\ldots,n$, where $0 \leqslant u_j \leqslant \bar{u}_j < \bar{p}_j/a_j$. For many resource allocation problems in physical or economic systems, however, the linear resource consumption function fails to reflect the law of diminishing marginal returns. This law states that productivity increases at a decreasing rate with the amount of resources employed. In order to model this, other studies of scheduling with resource allocation assume that the job processing time is a convex decreasing function of the amount of resources allocated to process the job. For a convex resource consumption function, the relationship between the job processing time and the resource allocated to the job is given by

$$p_j(u_j) = \left(\frac{w_j}{u_j}\right)^k. \quad (1)$$

This resource consumption function has been extensively used in continuous resource allocation theory (e.g., Armstrong, Gu, & Lei, 1997; Monma, Schrijver, Todd, & Wei, 1990; Scott & Jefferson, 1995; Shabtay, 2004).

Panwalkar and Rajagopalan (1992) are the first researchers to consider the single-machine earliness–tardiness scheduling problem with due date assignment and a linear resource consumption function under the condition that the compression rates of the jobs are $a_j = 1$. They reduce the problem $1|lin, a_j = 1, CON|\sum_{j=1}^{n}(\alpha E_j + \beta T_j + v_j u_j)$ to a linear assignment problem, implying that the problem is solvable in $O(n^3)$ time. The same approach to reducing a scheduling problem to a linear assignment problem and solving it in $O(n^3)$ time is adopted by Cheng, Oguz, and Qi (1996) and Biskup and Cheng (1999) in their extensions of the results by Panwalkar and Rajagopalan (1992). Cheng et al. (1996) deal with the problems $1|lin, a_j = 1, CON|\sum_{j=1}^{n}(\alpha E_j + \beta T_j + \gamma d + v_j u_j)$ and $1|lin, a_j = 1, SLK|\sum_{j=1}^{n}(\alpha E_j + \beta T_j + \gamma d_j + v_j u_j)$. Biskup and Cheng (1999) consider the problem $1|lin, a_j = 1, CON|\sum_{j=1}^{n}(\alpha E_j + \beta T_j + \theta C_j + v_j u_j)$ with $\theta \geqslant 0$. Alidaee and Ahmadian (1993) extend the results by Panwalkar and Rajagopalan (1992) to the case with identical parallel machines and solve the problem by reducing it to a transportation problem. For the common due date, Biskup and Jahnke (2001) present $O(n\log n)$ algorithms to minimize functions $f(u) + \sum_{j=1}^{n}(\alpha E_j + \beta T_j + \gamma d_j)$ and $f(u) + \sum_{j=1}^{n}(wU_j + \gamma d_j)$ under the condition of jointly reducible processing times, i.e., when $a_j = \bar{p}_j$ and $u_j = u$ for $j = 1,2,\ldots,n$, where $f(u)$ is a function of the resource consumption cost, which is the same for all the jobs, w is the unit penalty cost for finishing a job late, and $U_j = 1$ if job J_j is tardy and $U_j = 0$ otherwise. For both linear and convex resource consumption functions, Shabtay and Steiner (2007b) propose $O(n)$ algorithms for the problems $1|lin/conv, CON|\sum_{j=1}^{n}(w_j U_j + \gamma d + v_j u_j) + \delta C_{\max}$ and $1|lin/conv, SLK|\sum_{j=1}^{n}(w_j U_j + \gamma d_j + v_j u_j) + \delta C_{\max}$, and an $O(n^4)$ algorithm for the problem $1|lin/conv, DIF|\sum_{j=1}^{n}(w_j U_j + \gamma d_j + v_j u_j) + \delta C_{\max}$. For the problem of minimizing $\sum_{j=1}^{n}(\alpha E_j + \beta T_j + \gamma d_j + v_j u_j) + \delta C_{\max}$, Shabtay and Steiner (2008) provide unified algorithms (applicable to the different (DIF), constant (CON), and slack (SLK) rules of due date assignment) that require $O(n^3)$ time and $O(n\log n)$ time for a linear and for a convex resource consumption function, respectively.

To the best of our knowledge, all the above papers considering due date scheduling and controllable processing times treat the delivery cost as either negligible or irrelevant. In other words, they focus on the machine scheduling problem, while ignoring the problem of scheduling job delivery. However, delivery cost is a significant element of the production cost, which depends not only on when jobs are processed but also when finished jobs are delivered. Incidentally, Hermann and Lee (1993) remark that a more realistic production model should include scheduling of both job processing and job delivery. Hermann and Lee (1993) consider a batch delivery problem where all the jobs have a given restrictive common due date with the objective of minimizing the sum of earliness penalty, tardiness penalty, and delivery cost. They provide a pseudo-polynomial dynamic programming algorithm to solve the problem. Chen (1996) also studies this problem in which the common due date is a decision variable. Hence he adds a due date penalty to the objective function and shows that the problem can be solved in $O(n^5)$ time. Yin, Cheng, Xu, and Wu (2012) extend the problem studied by Chen (1996) to the case where an additional rate-modifying activity is allowed. The objective is to find a common due date for all the jobs, a location of the rate-modifying activity, and a delivery date for each job to minimize the sum of earliness, tardiness, holding, due date, and delivery cost. They provide some properties of the optimal schedule for the problem and present polynomial algorithms for some special cases. Hamidinia, Khakabimamaghani, Mazdeh, and Jafari (2012) consider a single-machine scheduling problem that involves earliness, tardiness, inventory cost, and batch delivery cost, which is shown to be NP-hard. They develop an integer programming approach and a genetic algorithm to solve it.

With a view to modelling a realistic production system, we consider in this paper the just-in-time scheduling that involves batch delivery cost, an assignable common due date, and controllable processing times simultaneously on a single machine, which extends the problem considered in Chen (1996) with controllable processing times. An example of a practical situation involving these three types of decision is as follows: consider a segment of