Computers & Industrial Engineering 63 (2012) 474–482

Contents lists available at SciVerse ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

An inventory model for deteriorating items under stock-dependent demand and controllable deterioration rate ☆

Yu-Ping Lee [a], Chung-Yuan Dye [b,*]

[a] Department of Business and Administration, Shu-Te University, Yen Chao, Kaohsiung 82445, Taiwan, ROC
[b] Graduate School of Business and Administration, Shu-Te University, Yen Chao, Kaohsiung 82445, Taiwan, ROC

ARTICLE INFO

Article history:
Received 26 July 2010
Received in revised form 14 January 2012
Accepted 17 April 2012
Available online 28 April 2012

Keywords:
Inventory
Stock-dependent demand
Deterioration
Partial backlogging
Preservation technology investment

ABSTRACT

In this paper, we formulate a deteriorating inventory model with stock-dependent demand by allowing preservation technology cost as a decision variable in conjunction with replacement policy. Moreover, it is assumed that the shortages are allowed and partially backlogged, depending on the length of the waiting time for the next replenishment. The objective is to find the optimal replenishment and preservation technology investment strategies while maximizing the total profit per unit time. For any given preservation technology cost, we first prove that the optimal replenishment schedule not only exists but is unique. Next, we show that the total profit per unit time is a concave function of preservation technology cost when the replenishment schedule is given. We then provide a simple algorithm to find the optimal preservation technology cost and replenishment schedule for the proposed model. Finally, we use some numerical examples to illustrate the model.

1. Introduction

In contrast to constant demand rate, for certain items, the displayed stock level has a positive impact on sales and profits. As pointed out by Levin, McLaughlin, Lamone, and Kottas (1972), “at times, the presence of inventory has a motivational effect on the people around it. It is the common belief that large piles of goods displayed in a supermarket will lead the customers to buy more”. Desmet and Renaudin (1998) also noted that impulsive buying categories have higher space elasticities, which is consistent with the interpretation that space has a causal effect on sales and not the converse. Due to the facts, a number of authors have developed the EOQ models that focused on stock-dependent demand rate patterns. Normally, there are two types of stock-dependent demand patterns. Gupta and Vrta (1986) assumed that the demand rate was a function of initial stock level. Baker and Urban (1988a, 1988b) considered a power-form inventory-level-dependent demand rate, which would decline along with the stock level throughout the entire cycle. Datta and Pal (1990) modified the model of Baker and Urban (1988b) by assuming that the stock-dependent demand rate was down to a given level of inventory, beyond which it is a constant. Goh (1994) relaxed the assumption of a constant holding cost in Baker and Urban (1988b). Later, Urban (1995) extended Datta and Pal’s (1990) model to allow shortages, where the unsatisfied demand is backlogged at a fixed fraction of the constant demand rate. Recently, Zhou, Min, and Goyal (2008) and Sajadieh, Thorstenson, and Jokar (2010) both studied the coordination of supply chain with power-form inventory-level-dependent demand. Hsieh, Dye, and Ouyang (2010) extended the models of Datta and Pal (1990) and Urban (1995) to allow shortages and time-dependent partial backlogging. In their model, the backlogging rate is considered to be a reciprocal of a linear function of the waiting time. Hence, the longer the waiting time is, the smaller the backlogging rate would be.

However, in many inventory systems, the deterioration of goods is a realistic phenomenon. It is well known that certain products such as refrigerated food, fruit and vegetable, fresh seafood and many others have a high deterioration rate. Mandal and Phaujdar (1989) developed a production inventory model for deteriorating items with uniform rate of production and linearly stock-dependent demand. Giri, Pal, Goswami, and Chaudhuri (1996) studied the model of Datta and Pal (1990) for deteriorating items. Meanwhile, Giri and Chaudhuri (1998) extended Goh’s model (1994) to consider the inventory model with a constant deterioration rate. Padmanabhan and Vrat (1995) considered an EOQ model for perishable items with a constant selling price and linearly stock-dependent demand. In particular, they assumed that the demand during the stock out period depends linearly on the inventory level. However, they ignored the lost sale cost in the formulation of the objective function since these costs are not easy to estimate, and its immediate impact is that there is a lower service level to customers. Based on the result of Padmanabhan and Vrat (1995) and

☆ This manuscript was processed by Area Editor: Alexandre Dolgui.
* Corresponding author.
E-mail address: chungyuandye@gmail.com (C.-Y. Dye).

0360-8352/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.cie.2012.04.006

Chung, Chu, and Lan (2000) developed the necessary and sufficient conditions of the existence and uniqueness of the optimal solutions of the total profit per unit time functions without backlogging and with complete backlogging. Dye and Ouyang (2005) extended the model of Padmanabhan and Vrat (1995) by proposing a reciprocal backlogging rate, and proved that the optimal replenishment policy not only exists but is also unique. Furthermore, the opportunity cost due to lost sales was taken account into the total profit. Chang, Goyal, and Teng (2006) then complemented the model of Dye and Ouyang (2005) for the situation that building up inventory is profitable.

Recently, Hou (2006), Hou and Lin (2006) and Jolai, Tavakkoli-Moghaddam, Rabbani, and Sadoughian (2006) incorporated effects of deterioration and stock-dependent demand rates to develop a finite time horizon inventory/production model under inflation. Soni and Shah (2008) proposed a deterministic inventory model with stock-dependent demand under trade credit policy. Lately, Yang, Teng, and Chern (2010) extended the model of Hou and Lin (2006) to allow shortages and time-dependent partial backlogging but with known selling price. Furthermore, because some of goods would have a span of maintaining quality, Wu, Ouyang, and Yang (2006) extended Dye and Ouyang's (2005) model for non-instantaneous deteriorating items. Chang, Teng, and Goyal (2010) amend Wu et al.'s model (2006) by changing the objective to maximizing the total profit and set a maximum inventory level in the model.

However, the deterioration rate of goods in the above mentioned papers is viewed as an exogenous variable, which is not subject to control. In practice, the deterioration rate of products can be controlled and reduced through various efforts such as procedural changes and specialized equipment acquisition. By the efforts of investing in reducing the deterioration rate, the retailer can reduce the economic losses, improve the customer service level and increase business competitiveness. Furthermore, the results of the sensitivity analysis in numerous studies showed that lower deterioration rate may be considered beneficial from an economic viewpoint. More recently, to express agreement with the practical inventory situation, Hsu, Wee, and Teng (2010) proposed a deteriorating inventory with a constant demand and deterioration rate. The main objective in their paper is to find the retailer's replenishment and preservation technology investment strategies which maximize the total profit per unit time. The graphical analysis approach is used to show the concavity of the objective function. However, the form of the preservation technology cost is a fixed cost per inventory cycle and this seems to be unrealistic. In real life, if new equipment, such as refrigeration units, is acquired, there will be a capital cost, which is often incorporated into models using an equivalent cost per period, or a leasing fee. Therefore, it should be a cost per period, not a fixed cost that is independent of the period length.

In this paper, to obtain robust and general results, we attempt to develop an inventory model with stock-dependent selling rate and preservation technology investment. In addition, the generalized time-dependent backlogging rate and productivity of invested capital are taken into the model. We also assume that the preservation technology cost depends on the cycle length. The objective is then to find the optimal replenishment and preservation technology investment strategies while maximizing the total profit per unit time. In the next section, the assumptions and notations related to this study are presented. Then, we prove that the optimal replenishment policy not only exists but is unique, for any given preservation technology cost. Next, we show that the total profit per unit time is a concave function of preservation technology cost when the replenishment schedule is given. We also provide a simple algorithm to find the optimal replenishment policy. Finally, some numerical examples are presented to illustrate the model and the sensitivity analysis of the optimal solutions with respect to parameters of the system is also carried out, which is followed by concluding remarks.

2. Notation and assumptions

2.1. Notation

To develop the mathematical model of inventory replenishment schedule, the notation adopted in this paper is as below:

A	the replenishment cost per order
C	the purchase cost per unit
S	the selling price per unit, where $S > C$
Q	the ordering quantity per cycle
L	the number of lost sales per cycle
i	the inventory carrying charge rate
C_2	the backorder cost per unit time
R	the opportunity cost (i.e., goodwill cost) per unit
t_1	the time at which the inventory level reaches zero
T	the length of the inventory cycle
θ	the deterioration rate, a fraction of the on-hand inventory
ξ	the preservation technology cost per unit time for reducing deterioration rate in order to preserve the products, where $0 \leqslant \xi \leqslant w$
w	the maximum capital constraint
$I(t)$	the level of inventory at time t
$\Pi(t_1, T, \xi)$	the total profit per unit time

2.2. Assumptions

In addition, the following assumptions are imposed:

1. The replenishment rate is infinite and lead time is zero.
2. The time horizon of the inventory system is infinite.
3. The demand rate function $D(t)$ is deterministic and a function of instantaneous stock level $I(t)$. When inventory is positive, $D(t)$ is given by:

$$D(t) = \alpha + \beta I(t), \quad 0 \leqslant t \leqslant t_1,$$

and when inventory is negative, $D(t)$ is given by:

$$D(t) = \alpha, \quad t_1 < t \leqslant T,$$

where $\alpha > 0$ and $0 < \beta < 1$ are termed as scale and shape parameters, respectively.
4. There is no repair or replacement of deteriorated units. The items will be withdrawn from warehouse immediately as they become deteriorated.
5. The reduced deterioration rate, $m(\xi)$, is an increasing function of the preservation technology cost ξ, where $\lim_{\xi\to\infty} m(\xi) = \theta$.
6. Shortages are allowed. The fraction of shortages backordered is a decreasing function $b(x)$, where x is the waiting time up to the next replenishment, and $0 \leqslant b(x) \leqslant 1$ with $b(0) = 1$. Note that if $b(x) = 1$ (or 0) for all x, then shortages are completely backlogged (or lost).

3. Model formulation

Using above assumptions, the inventory level follows the pattern depicted in Fig. 1. The depletion of the inventory occurs due to the combined effects of the demand and deterioration in the interval $[0, t_1)$and the demand backlogged in the interval $[t_1, T)$. Hence, the variation of inventory level, $I(t)$, with respect to time can be described by the following differential equation: