



Inventory models for multi-warehouse systems under fixed and flexible space leasing contracts[☆]

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ARTICLE INFO

Article history:

Received 1 May 2010

Received in revised form 2 March 2011

Accepted 12 May 2011

Available online 20 May 2011

Keywords:

EOQ
Multi-storage facilities
Ordering quantity
Contract
Spot market

ABSTRACT

In this paper, we analyze a practical situation in which an inventory manager is faced with several options to store excess stocks whenever the storage capacity of his/her warehouse is insufficient. The manager can choose from either storage space providers through fixed long term or flexible leasing contracts, or the manager can acquire the extra required space from the spot market. We formulate this inventory problem with multiple storage facilities as a nonlinear program and show that it has a global optimal solution. We then provide closed-form solutions for the optimal ordering quantity and leased spot market space depending on the value of the unconstrained economic order quantity. In addition, we develop some structural properties for the optimal ordering policy and include several examples to illustrate the formulated models.

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1. Introduction

Nowadays, management can no longer afford warehouses with large storage capacities due to the increase in land acquisition costs. Induced by this lack of storage space, inventory managers are coerced into turning down supplier discounts. An option for inventory managers to take advantage of these attractive price discounts associated with large orders, is to store the excess inventory in rented warehouses. It is a common practice to acquire more space on a rental basis whenever the storage capacity of the owned warehouse is insufficient. In such situations, the owned warehouse (OW) is utilized to its maximum capacity and the surplus stocks are transferred to be accommodated in a rented warehouse (RW).

The economic order quantity (EOQ) model and its variants have been studied extensively in the inventory literature. Extensive research has been undertaken to extend these basic inventory models in terms of relaxing some of their assumptions so that they can conform more closely to practical situations. One of the restrictive assumptions of the EOQ model is the unlimited storage capacity of the owned warehouse. However, as mentioned above, it can be economically beneficial to order quantities larger than the internal storage capacity and resort to external storage facilities to stock the excess quantities.

Due to different storage conditions, the inventory holding rate in RW is generally higher than those in OW. Therefore, in order to reduce inventory holding costs, customers are first served from

stocks that are stored in RW. Once these stocks are depleted, the stock in OW is used to meet the remaining customer orders. This type of storage system follows the *last-in first-out* order filling approach and assumes that the stocked items are not perishable. It is also assumed that external storage facilities are located near the owned warehouse so that transportation costs between warehouses are not significant and can thus be neglected in the analysis. Alternatively, we assume that customers can be served directly from external space providers.

Hartley (1976) was the first to develop an EOQ model for a two-warehouse inventory system. Later, Sarma (1983) generalized Hartley's model by including transportation cost into the model and proposed a new ordering rule. Sarma (1987) also extended Hartley's model to include the effects of deterioration in both warehouses. Goswami and Chaudhuri (1992) extended Sarma's model (1983) further by considering linearly time varying demand. Several other authors studied the two-warehouse inventory system under different settings dealing with deteriorating items, time value of money, inflation, shortages, stock level-dependent demand, finite time horizon, and finite production rate (see for example Bhunia & Maiti, 1998; Hsieh, Dye, & Ouyang, 2008; Kar, Bhunia, & Maiti, 2001; Pakkala, 1992; Pakkala & Achary, 1992; Yang, 2004; Zhou & Yang, 2005). Very recently, Chung, Her, and Lin (2009) extended Salameh and Jaber's model (2000) of a single warehouse system with defective items to a two-warehouse inventory system.

All of the above referred research works assumed that the rented warehouse is equipped with abundant space. However, given that the rented warehouse is serving more than one customer, this assumption is impractical. Therefore, this paper assumes that

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the rented warehouse, also, has a limited storage capacity. In case the ordering quantity is larger than the sum of owned and rented capacities, then extra space can be acquired from spot markets. Logistics companies have recently started to acquire storage space from spot markets as an alternative to the traditional long-term fixed contracts (Chopra & Meindl, 2010). Spot markets are usually used as the last recourse as they entail higher space leasing rates, which are charged on the actual space used and not on the maximum space needed.

The remainder of this paper is organized as follows; in the next section, we present the notations and assumptions used to develop the proposed inventory models. In the consecutive two sections, we develop the mathematical models for the fixed and flexible renting contracts, respectively. Finally, the last section concludes the paper.

2. Notations and assumptions

The following notations will be used throughout the paper when developing the mathematical models.

D	demand per unit time
A	fixed ordering cost per order
h_o	holding cost per unit per unit time in the owned warehouse
h_c	holding cost per unit per unit time in the rented warehouse with a fixed contract
h_{cf}	holding cost per unit per unit time in the rented warehouse with a flexible contract
h_s	holding cost per unit per unit time in the spot market warehouse
r_c	renting cost per unit per unit time in the rented warehouse with a fixed contract
r_{cf}	renting cost per unit per unit time in the rented warehouse with a flexible contract
EOQ	economic order quantity = $\sqrt{2AD/h_o}$
M	maximum space that can be rented in the warehouse with a fixed contract
M_1	minimum space that is eligible to be rented for a flexible renting rate
M_2	maximum space that can be rented in the warehouse with a flexible contract

Additional notation shall be introduced later when required.

The mathematical models developed in this paper are based on the following assumptions.

- (1) Maximum of W units can be stored in the owned warehouse.
- (2) In case of a two-warehouse system, items in the OW are used to satisfy customer demand only after inventory in the RW is completely depleted.
- (3) In case of a three-warehouse system, customer orders are first met from stocks stored in the spot market facility until it is emptied, then from items present in the RW. Once these stocks are completely depleted, the remaining order quantity is satisfied from the OW.
- (4) The transportation costs between warehouses are negligible.
- (5) The unit holding cost per unit time in the RW does not include the renting cost.
- (6) The unit holding cost per unit time in the spot market facility covers the renting cost as the latter is charged for each unit stored per unit time and it is not based on the maximum space reserved.
- (7) The unit holding cost per unit time in the RW is larger than the one in the OW.

- (8) The unit holding cost per unit time in the spot market facility is larger than the one in the RW.
- (9) The product does not deteriorate while in storage.

3. Fixed renting contract model

Consider a situation, where an inventory manager must decide whether to sign a long-term fixed renting space contract with a third-party warehouse and/or obtain the needed space from the spot market every year. By selecting the fixed contract, the inventory manager is committed to a fixed storage capacity of M units. The long term lease cost is $\$r_c$ per unit per year and the spot market storage rate is estimated at $\$r_s$ per unit per year. If the required extra space (space required to store stock when the OW has reached its maximum capacity) is smaller than the space agreed with the third party warehouse, the inventory manager has to pay the renting cost of all M units. On the other hand, if this space is larger than M units, extra space will be provided from the spot market. The inventory level variation over one cycle for each of these two cases is shown in Fig. 1.

Based on the shape of the areas in Fig. 1, the holding cost per cycle in the owned warehouse is h_o times the area of the lower trapezoid with parallel sides equal to Q/D and $(Q - W)/D$. It is given by

$$\frac{h_o}{2} W \left[\frac{Q}{D} + \frac{Q - W}{D} \right],$$

which can be rewritten as

$$\frac{h_o}{2D} [Q^2 - (Q - W)^2].$$

The holding cost per cycle in the rented warehouse is h_c times the area of the triangle of Fig. 1a in case $(Q - W) < M$ (the spot market option is not used) and h_c times the area of the second trapezoid of Fig. 1b with parallel sides $(Q - W - M)/D$ and $(Q - W)/D$ in case $(Q - W) > M$. Based on this observation, the holding cost per cycle is expressed mathematically as

$$\frac{h_c}{2D} [Q - W + \text{Max}(0, Q - W - M)] \text{Min}(M, Q - W).$$

Note that when $\text{Max}(0, Q - W - M)$ is zero and $\text{Min}(M, Q - W)$ is $(Q - W)$, then the spot market space is not needed and the last mathematical expression is based on the area of the triangle in Fig. 1a. On the other hand, when $\text{Max}(0, Q - W - M)$ is $(Q - W - M)$ and $\text{Min}(M, Q - W)$ is M , then the holding cost per cycle in the rented warehouse is h_c times the area of the second trapezoid of Fig. 1b.

Letting $x = \text{Max}(0, Q - W - M)$, the space leased in the spot market, and $y = \text{Min}(M, Q - W)$, the space rented as per the fixed contract, the above holding cost per cycle in the rented warehouse can be written as

$$\frac{h_c}{2D} [Q - W + x]y.$$

Next, given that

$$y = \text{Min}(M, Q - W) = Q - W - \text{Max}(0, Q - W - M) = Q - W - x,$$

the holding cost per cycle in the rented warehouse can be rewritten as

$$\frac{h_c}{2D} [(Q - W)^2 - x^2].$$

The holding cost per cycle in the spot market facility is h_s times the area of the top triangle in Fig. 1b, that is,

$$\frac{h_s}{2D} x^2.$$

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