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Dynamic ordering and pricing for a perishable goods supply chain ☆

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ABSTRACT

The paper studies the combined problem of pricing and ordering for a perishable product supply chain with one supplier and one retailer in a finite horizon. The lifetime of the product is two periods and demand in each period is random and price-sensitive. In each period, the supplier determines first a wholesale price and then the retailer decides an order quantity and retail prices. We show that the optimal pricing strategy for the non-fresh product depends only on its inventory, and the optimal pricing strategy and the optimal order quantity for the fresh product depend only on the wholesale price and they have a constant relation. Moreover, the game between the retailer and the supplier for finite horizon is equivalent to a one period game with only one order. Thus, the optimal policies are identical at each period. For the additive and multiplicative demands, we further obtain equations to compute the optimal strategies. All of above results are extended into the infinite horizon case and longer lifetime products. Finally, a numerical analysis is given.

1. Introduction

Some goods are perishable because of physical deterioration, such as blood, fresh vegetable, and meats. Some are because their values deteriorate, such as berth in plane, a bed in hotel, and a ticket in cinema. Some other are because of technological changes and shifts in consumer preferences. So, most firms dealing with perishable goods need to price and order everyday, which is a multi-period decision problem in finite or infinite horizon. For a perishable product supply chain, it becomes more important to study how to control inventory and how to decide prices. This paper considers the combined problem of pricing and ordering of a retailer and pricing of a supplier for a perishable product in framework of the supply chain. The demand in each period is random and price-sensitive, and inventory levels are reviewed periodically. Products with different remaining lifetime have different retail prices. In each period, the supplier determines first a wholesale price, and then the retailer decides his order quantity for the fresh goods and retail prices for products with different lifetimes. Both the supplier and the retailer want to maximize their expected total profits, respectively.

There is rich literature about perishable inventory. Fries (1975) shows that the optimal ordering policies depend on the age distribution of the inventory on hand, and implementing an optimal policy is impractical as the length of the product lifetime increases and the state vector grows. This motivates lots of further studies. The policy that orders are placed at the end of each period to bring the inventory to a specific level is proposed by Cohen (1976), Nahmias (1976) and Chazan and Gal (1977). Cooper (2001) analyzes this policy by studying pathwise properties and performance bounds for a perishable inventory system. Liu and Lian (1999) analyze a (s,S) continuous review inventory system with perishable goods and instantaneous replenishment. Using Markov renewal approach, they obtain closed-form solutions for the steady state probability distribution of the inventory level and system performance. Goto, Lewis, and Puterman (2004) study the optimal ordering policy for the airline meal by Markov decision process. The policy can adjust the meal order quantity to match the passenger load based on analyzing actual data. Goyal and Giri (2001) give a complete review of the research on the inventory of perishable goods.

About the combined pricing and ordering problems for perishable products, there are only several models developed in the literature with demand being either deterministic or random with known distribution. Rajan, Steinberg, and Richard (1992) and Abad (1996) develop continuous time pricing and ordering policies for deterministic demand. Gallego and van Ryzin (1994) develop a pricing model for a perishable product with random demand and no inventory replenishment. Burnetas and Smith (2000) consider the combined problem of pricing and ordering for a perishable product with uncertain price-sensitive demand. The retailer can only observe sales of the product and must decide the price and lot size in every period based on the previous prices, lot sizes, and sales levels. They find the optimal order quantity first and then

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the optimal price. But they assume that no inventory is carried from one period to the next. Chun (2003) assumes that the customer's demand is a negative binomial distribution, in a pricing and ordering problem where a seller must determine prices for several units of a perishable product to be sold for a limited period of time. He determines the optimal price based on the demand rate, buyers' preferences, and length of the sales period. Bhattacharjee and Ramesh (2000) present a multi-period inventory and pricing model for a single product with a fixed lifetime and deterministic demand. They assume that order quantity is determined after the price is determined. The problem is modeled as a dynamic program, and it is shown that the maximum profit function is continuous piecewise concave. Bisi and Dada (2007) study the optimal dynamic ordering and pricing policies for a retailer with a mixed (additive and multiplicative) price-sensitive demand. They formulate the problem as a Bayesian Markov decision process by introducing the stocking factor, derive the optimal ordering and pricing policies as functions of the stocking factor, and show that when lost sales are unobservable, the optimal stocking factor with perishable inventory is always at least as large as the one for the single-period model.

Another literature related to this paper is about perishable products' life-cycle. Nahmias (1977) studies inventory control of a perishable product when both demand and lifetime are random. Nahmias (1982) gives a survey and classifies perishability into two types: fixed lifetime and random lifetime. Lian, Liu, and Neuts (2005) consider a discrete-time (s,S) inventory model where items have a random lifetime with a discrete phase-type distribution and demands arrive in batches following a discrete phase-type renewal process. They compare the results with those for the constant lifetime and conclude that the variance of the lifetime significantly affects the system behavior. Webster and Weng (2000) study the problem that a manufacturer sells a short life-cycle product to a single risk neutral retailer. The fixed lifetime is called by van Zyl (1964) as 'age dependent' perishability. Chen and Chen (2005) consider a manufacturer who produces and sells a single product that is subjected to continuous decay over a lifetime. The demand is price-dependent and time-varying, shortages are completely backlogged, and the objective is to determine price and production lot-size/scheduling for maximizing the total profit over multi-period planning horizon. Li, Lim, and Rodrigues (2009) study combined pricing and inventory control in an infinite horizon for a perishable product with a fixed lifetime and inventory is consumed in a first-in-first-out manner.

A recent research area on ordering and pricing is under the framework of supply chains. Webster and Weng (2008) study a supply chain with a manufacturer and a distributor operating to meet price sensitive random demand for products with short life cycles under two scenarios. In the manufacturer-controlled scenario, the distributor shares information on price-sensitive random demand with the manufacturer and the manufacturer controls the supply chain stocking decisions and bears the risk of overstocking costs. The distributor-controlled scenario works in the opposite direction. They suggest that a control scenario requiring explicit information sharing from one party to another may leave one party exploited and might result in an outcome that is only beneficial to one party. Hsieh, Wu, and Huang (2008) examine coordinated decisions in a decentralized supply chain that consists of one supplier and one retailer facing random demand of a product with short life cycle. They develop three coordinated models in contrast with the basic and uncoordinated model. Both papers above are concerned on one period.

In this paper, we study the periodically ordering and pricing problem in the framework of a supply chain. We show that for the two-period lifetime product the finite or infinite horizon game between the retailer and the supplier is equivalent to a one period game with only one order. Thus, the optimal policies are identical at each period, and (a) the optimal pricing strategy for non-fresh product depends only on its initial inventory of the current period; and (b) the optimal pricing strategy and the optimal order quantity for fresh product, depending only on the wholesale price, have a constant relation. Moreover, for the additive and multiplicative demands, we obtain equations to compute the optimal strategies. The results are extended into the case with longer lifetime products.

The remainder of the paper is organized as follows. In Section 2, the model and notations are presented. In Section 3, the optimal policies are discussed for finite horizon. In Section 4, the results are extended into the infinite horizon case and longer lifetime case. In Section 5, a numerical analysis is given. Section 6 is a concluding section.

2. The problem with 2 days lifetime

The problem we studied is described as follows. One supplier and one retailer trade a perishable product. The fresh goods can be sold for two days, and those not sold in the first day can be sold at the next day. We call the fresh goods as product 2 and that can be sold only for one day as product 1. Unsold product 1 will be thrown away. That is, the retailer does not pay holding cost for the unsold product 1, but he has to be charged with penalty cost for them. We introduce situations on the supplier, the retailer, and demand faced by the retailer one by one as follows.

(1) The supplier produces product 2 with a constant unit cost c. For the perishable characteristic of the product and the demand uncertainty, the ordering is made everyday and thus the supplier has to determine a wholesale price w everyday.
(2) The retailer orders only the fresh goods one time each day according to his demand, inventory, and the supplier's wholesale price. One reason is that he has to provide consumers some fresh goods everyday, and the other is that product 1 is so easy to be perished that there is much risk of paying penalty cost. To product 2, if some of them can not be sold out, they will be stored and then result in holding cost that the retailer must pay. So the retailer has to determine how much he orders to balance the inventory. We denote by u the unit penalty cost, h the unit holding cost, and q the order quantity of the retailer for product *2* (i.e., the fresh product). Another more important decision for the retailer is to make retail price p_k for product $k = 1, 2$.
(3) Demand faced by the retailer is random and price-sensitive. Although there are lots of various factors affecting the demand, we focus our attention to effect of retail price on demand. Let $D_k(p_k)$ be demand for product k in one period, when the retail price for product k is p_k, $k = 1, 2$.

The objectives of the supplier and the retailer are to maximize their total expected profits, respectively. We assume a discount factor $\beta \in (0,1]$. How could the retailer determine his optimal ordering and pricing policy and how could the supplier find the optimal wholesale price? We will answer these questions in the following sections.

We study the problem described above in the next section for finite horizon, and in Section 4 for infinite horizon and longer lifetime.

3. Optimal policies for finite horizon

3.1. Optimal equation and optimal policies

Suppose that the wholesale price decided by the supplier is w for the fresh product and the inventory of the product 1 is x. Then,