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A study on limited-cycle scheduling problem with multiple periods ☆

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ABSTRACT

This paper considers the optimal arrangement to minimize the total expected risk on limited-cycle scheduling problem with multiple periods. In this paper, first, the multi period problem is systematically classified and modeled. Next, a recursive formula for the total expected risk is presented, and an algorithm for optimal assignments in limited-cycle problems with multiple periods is proposed based on the branch and bound method. In addition, the effectiveness of the proposed algorithm is shown by investigating the calculation time of the computer. Finally, the optimal assignments are studied by numerical experiments and the property useful for the design of a production seat system is found.

1. Introduction

Scheduling problems concern the allocation of limited resources over time among both a manufacture period and multiple periods. The allocation also affects the optimality of a schedule with respect to criteria such as production cost, idle risk and delay risk. Under the condition of uncertainty, the result or efficiency of a period is often controlled not only by the risks of this period but also by the risks generated beforehand. Whether the process (period or seat) satisfies the due time (restriction) usually depends on the state of the past process, as seen in Wight (1974), Verzijl (1976) and Benders (2002). In particular, in the case of the risk which depends on a situation of a past process (for instance, the case of a manufacture line for a multi period), which assignment of machines, workers or jobs is most efficient and economical (optimal assignment) is an important problem in load/risk planning (for example, see Bergamashi, Cigolin, Perona, & Portiol (1997) & Swamidass (2000)).

As limited-cycle scheduling problems, Verzijl (1976) analyzed the element and construction of the production system. Enns (2001) presented a framework for the analysis of delays within the production system. Benders (2002) gave a review for the origin and solution of period batch control system. Wu and Zhou (2008) concerned with the problem in scheduling a set of jobs associated with random due dates on a single machine so as to minimize the expected maximum lateness in stochastic environment. Recently, Xia and Wu (2005) presented an easily implemented hybrid algorithm for the multi-objective flexible job-shop scheduling problem. Zhiqiang, Shuzhen, Guangjie, and Guangyu (2009) proposed a new hierarchical scheduling algorithm to solve the complex product flexible scheduling problem with constraint between jobs.

We consider "limited-cycle model with multiple periods" (Yamamoto, Matsui, & Liu, 2006). In this model, there exists an object with some constraints, and if the object does not satisfy constraints, this becomes a risk and the object occurs repeatedly for multiple periods (or processes). This model is applied to manufacture lines, time-bucket balancing, production seat systems and so on (Matsui, 2004, 2005, 2008). In the case of manufacture lines, a production time as the object, the due time of production time as the constraints, and idle or delay time as the risk are considered.

In this paper, we consider limited-cycle scheduling problem with multiple periods, where we find out assignment (optimal assignment) of the objects with minimum expected risk (or cost) in the limited-cycle model with multiple periods. For obtaining optimal assignment, we should obtain the expected costs for all the assignments and we need exhausted time. So, Yamamoto et al. (2006) proposed the effective algorithm for the optimal assignment but we need an more effective algorithm, which has shorter calculation time for finding out the optimal assignment. Therefore, The purpose of this paper is (1) to propose the effective

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algorithm for the optimal assignment in limited-cycle scheduling problem with multiple periods and (2) to research the property of the optimal assignments by using the proposed algorithm.

The organization of this paper is as the follows. In Section 2, we describe the limited-cycle model with multiple periods in detail and define limited-cycle scheduling problem with multiple periods. Next, in Section 3, we propose a recursive formula for the total expected cost and an algorithm for optimal assignments aimed at the minimum total expected cost, based on the branch and bound method. We performed a numerical experiment in order to compare our proposed algorithm with the existing algorithm. By the experiments, we show the effectiveness of the proposed algorithm. Finally, in Section 4, we research the optimal assignments by numerical experiments. From the results of the numerical experiments, it is found out that there exists a similar property to the "Bowl phenomenon" in a series manufacturing line (Hillier & Boling, 1966; Magazine & Silver, 1978, recently Hillier & Hillier, 2006.) with simple assumptions. The results obtained in this paper are useful for the design of a production seat system.

2. Multi limited-cycle model

Before we describe multi limited-cycle model in detail, for an example, we consider a manufacture line as shown in Fig. 1. the manufacture line has n processes and each process is executed at one period. A product is produced through the manufacture line. First, a material is processed at process 1 (P_1, process is executed at Period 1), the processed material is at process 2 (P_2, Period 2), at process 3 (P_3, Period 3) and at process n (P_n, Period n) in turn (Matsui, Shingu, & Makabe, 1977). Now, we consider the following situation. The limited production time (Z), such as target production time or cycle time, is given for n periods. In this situation, delay or idle in each period occurs. When the delay occurs, this usually influences the following periods. We describe this with not times but costs, because the delay can be recovered by the overtime work or spare workers in each period. As the delay cost, we consider the cost per unit time ($C_l^{(k)}$), which occurs in the period if delay occurs in consecutive k periods before its period. When the idle occurs, we consider the cost per unit time (C_s), because stocks of the products will be generated. Furthermore, we consider also the cost per unit time (C_t) for the limited production time (Z) as a fixed cost. Fig. 2 shows the above costs in the model. In Fig. 2, C_s occurs for the idle at period 1, $C_l^{(1)}$ occurs for the delay at period 2 (i) as idle occurs at period 1 ($i-1$), and $C_l^{(2)}$ occurs for the delay at period $i+1$ as the idle occurs at period $i-1$ and the delay at period i occurs.

Based on the above situation, we consider the model of the multi limited-cycle model in following assumptions:

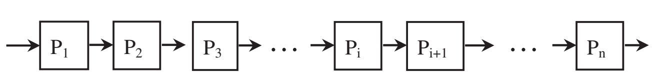

Fig. 1. A manufacture line.

(1) n is the number of periods (it may be considered that n is the number of production seats or production processes).
(2) The prescribed limited production time (or target production time or cycle time) is denoted by Z.
(3) The production time of one period is denoted by T and is assumed to be exponentially distributed. The production times are assumed to be statistically independent.

Then, we suppose the following costs in the multi limited-cycle model as shown in Fig. 2.

(4) The cost per unit time (C_t) for the production time limit (Z) occurs in each period.
(5) When $T<Z$, the idle cost per unit time (C_s) occurs in each period.
(6) When $T>Z$, the delay cost per unit time ($C_l^{(k)}$) occurs in the period if delay (that is, $T>Z$) occurs in consecutive k periods before its period, for $k=1,2,\ldots,n$, where $C_l^{(1)} \leqslant C_l^{(2)} \leqslant \cdots \leqslant C_l^{(n)}$.

Now, we consider the situation in which one worker is assigned to each period in the above multi limited-cycle model. One of the most important problems is how to assign workers to periods for minimizing the expected cost in n periods. We call such a problem the optimal assignment problem. For stating the optimal assignment problem, we define the following notations:

W: the set $\{\mu_1,\mu_2,\ldots,\mu_n\}$, where μ_l is the mean processing rates of worker l ($l=1,2,\ldots,n$), and $\mu_1 \leqslant \mu_2 \cdots \leqslant \mu_n$.
π: the permutation composed from 1 to n, where $\pi=(\pi(1),\pi(2),\ldots,\pi(n))$.
S_n: the set of π, where $\pi(i)$ is the number of workers assigned in period$i$, for example, when $n=5$, $\pi=(2,4,1,5,3)$ means the case in which the workers with μ_2, μ_4, μ_1, μ_5 and μ_3 are assigned in periods 1–5, respectively. In this paper, permutation π is called assignment π because π denotes the assignment of workers.
$TC(n;\ \pi,W)$: the total cost in periods $1-n$ when workers' mean processing rates are given by W and workers are assigned in periods 1 to n by assignment π.

By using these notations, the optimal assignment problem with multiple periods becomes the problem of obtaining assignment π^* in the following equation (Yamamoto, Matsui, & Bai, 2007):

$$TC(n;\pi^*,W) = \min_{\pi \in S_n} TC(n;\pi,W). \tag{1}$$

3. Algorithm of optimal assignment using branch and bound

This section describes our proposed algorithm for the optimal assignment. Section 3.1 describes the theorem and property of the algorithm, and Section 3.2 describes the basic idea and the procedure of the algorithm. Finally, Section 3.3 describes the evaluation of the algorithm with numerical experiments.

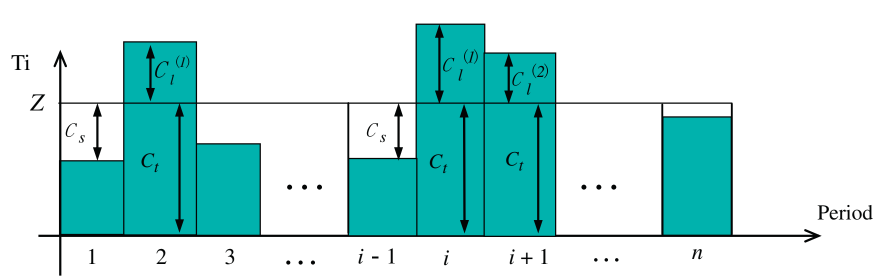

Fig. 2. Costs in the model.